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Cognitive Barriers in the Transition from Graphical to Symbolic Representation of Integral Concepts: A Systematic Literature Review

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Abstract

Integral calculus requires students to coordinate graphical, symbolic, and verbal representations rather than rely only on procedural manipulation. However, many students experience cognitive difficulty when converting graphical information, such as shaded regions or curves, into formal integral notation. This study synthesizes empirical evidence on cognitive barriers in the graphical-to-symbolic transition of integral concepts through a Systematic Literature Review (SLR). The review followed PRISMA 2020 guidance and examined empirical journal articles and conference proceedings published from 2016 to 2025 at the university or pre-university transition level. Records were searched in Google Scholar, Garuda, ERIC, and ScienceDirect; after screening and eligibility assessment, eleven studies were included. The selected studies were analyzed through descriptive synthesis and thematic coding to identify literature distribution, theoretical tendencies, and recurring barrier types. The synthesis indicates that representation-oriented and cognitive frameworks, including Duval's Semiotic Register Theory, APOS theory, mental models, and accumulation perspectives, are frequently used to interpret students' difficulties. Three central barriers were identified: (1) visual attribute disorientation, in which students misread visually salient features such as intercepts as integration limits; (2) cognitive conflict in signed area, in which students struggle to reconcile positive area images with negative symbolic accumulation; and (3) multicurve decomposition barriers, in which students have difficulty determining boundaries, intersections, and upper-lower function relationships. These findings suggest that graphical-to-symbolic transition is not merely a procedural issue but a representational coordination challenge. The review recommends barrier-specific learning designs that combine graphical tasks, dynamic visualization, accumulation-based reasoning, and structured error reflection.

Keywords: Integral calculus; Cognitive barriers; Multimodal representation; Semiotic transition; Systematic literature review

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INTRODUCTION

Integral calculus has long been recognized as a fundamental pillar of the college mathematics curriculum, serving as a bridge connecting the concepts of accumulation, limits, and geometry. A comprehensive and in-depth understanding of integrals requires students not only to master calculation procedures but also to flexibly operate across multiple modes of representation: graphical representation (visual, such as the area under a curve), symbolic representation (formal mathematical notation), and verbal representation. The ability to represent and translate mathematical ideas across these modes is a key indicator of strong conceptual understanding (Janvier, 1987; Duval, 2006; Mainali, 2021). In the landscape of mathematics education in Indonesia, mastery of integral concepts integrated with these various representations is becoming increasingly crucial as the curriculum demands developing higher-order thinking skills and applied understanding.

The theoretical foundations of mathematics learning place the process of translation between representations as a complex and essential cognitive activity. When students are faced with integral problems, they are expected to be able to coordinate representations, for example, translating visual information from a graph (such as the area under a curve) into the formal structure of a definite integral, and vice versa. This ability reflects an integrated understanding, where students not only memorize formulas but also understand the meaning behind the symbols they manipulate. This cognitive process, as described in representation theory, involves identifying a mathematical object from one mode, deconstructing its elements, and reconstructing it in a new mode while maintaining the same meaning.

In the context of higher education in Indonesia, integral learning still faces significant pedagogical challenges. Although integral concepts are a core topic taught in various study programs, classroom learning practices are often more oriented towards algorithmic procedures and calculations. Many students are accustomed to calculating integrals using standard formulas, but struggle when faced with non-routine problems that require graphical interpretation. A key challenge that often arises is a cognitive "gap" when students attempt to translate visual information (e.g., the shaded area under a curve) into the correct definite integral notation. Students may be able to calculate the integral value but fail to construct the appropriate symbol based on the graphical stimulus. This contextual obstacle, which can be influenced by spatial abilities, understanding of functions, and misconceptions about notation, underscores the importance of systematically examining how cognitive barriers in this graphical-symbolic transition $\int_a^b f(x)dx$ occur.

Recent studies over the last decade have consistently reported that students experience persistent difficulties in understanding definite integrals, particularly in interpreting area, determining limits of integration, coordinating graphical and symbolic representations, and reasoning about accumulation. These difficulties have been documented in technology-supported learning environments, mental model studies, error analysis, and representational tasks (Tatar & Zengin, 2016; Greefrath et al., 2021; Stevens & Jones, 2023; Uripno et al., 2024; García-García et al., 2025; Romero Osorio et al., 2025). Classical works such as Janvier (1987), Tall and Vinner (1981), and Duval (2006) remain relevant as theoretical foundations for understanding representational coordination and concept image, but the present review prioritizes empirical evidence published within the last ten years.

Other studies highlight the role of representation in integral understanding and how the use of technology can help bridge the gap between graphical and symbolic representations. Despite these important contributions, the results of this research remain scattered and incompletely mapped. However, previous studies have tended to discuss students' difficulties in definite integrals as general errors, misconceptions, or intervention-related outcomes. They have rarely organized these difficulties into a coherent typology of barriers that explains why students fail to translate graphical information into symbolic integral notation. In particular, the literature has not sufficiently distinguished among three interrelated barriers: conceptual barriers, related to students' understanding of area, accumulation, function, and limits; representational barriers, related to students' ability to coordinate graphical and symbolic modes; and procedural-symbolic barriers, related to constructing correct integral notation, including integrand, bounds, and differential notation. This lack of typological synthesis makes it difficult to identify which barriers are most dominant, how they interact, and what instructional responses are needed. Consequently, a comprehensive understanding of the research landscape related to these specific cognitive barriers, particularly in the context of higher education, remains limited.

This gap underscores the need for a systematic review that can synthesize empirical findings, highlight common patterns of cognitive barriers, and identify their underlying causes from a cognitive theory perspective. A structured mapping of previous studies is crucial to clarify the types of difficulties experienced by students, the most relevant theoretical frameworks for analyzing them, and their practical implications for pedagogical development and instructional design. This will be highly beneficial for lecturers, researchers, and curriculum developers in designing more effective learning strategies to address these cognitive barriers.

Therefore, this study aims to provide a systematic review of the body of research on cognitive barriers in the transition from graphical to symbolic representation of integral concepts. The purpose of this study is to identify the types of cognitive barriers experienced by students and analyze the causes of these misconceptions based on the perspective of cognitive theories in the literature. The novelty of this review lies in its use of a three-barrier typology to synthesize previous findings on students' cognitive difficulties in translating graphical representations of integrals into symbolic notation. Rather than merely listing common errors or summarizing intervention studies, this review classifies the barriers into conceptual, representational, and procedural-symbolic dimensions. This typology enables a more systematic explanation of how students' difficulties emerge, how they are theoretically grounded, and how they may inform instructional design in calculus learning. Furthermore, this review provides practical contributions in the form of insights that can inform instructional design, material development, and calculus course strategies that are more responsive to students' cognitive needs.

Based on the identified gap, this review is guided by the following questions: (1) What are the distribution and theoretical trends in studies on cognitive barriers in integral representation?; (2) How can students' cognitive barriers in translating graphical representations into symbolic integral notation be classified into conceptual, representational, and procedural-symbolic barriers?; and (3) What cognitive and pedagogical factors explain the emergence of these three types of barriers? By formulating this problem, this introduction lays the groundwork for a systematic review of studies related to cognitive barriers in integral learning. At the same time, a detailed interpretation of the study results will be presented in the Discussion section to maintain a clear structure.

METHOD

This study employed a Systematic Literature Review or SLR approach to identify, evaluate, and synthesize empirical findings concerning students' cognitive barriers in the transition from graphical to symbolic representations of integral concepts. The SLR approach was selected because it provides a systematic, transparent, and replicable procedure for collecting, screening, evaluating, and synthesizing research evidence. Through this approach, the study aims to map the conceptual development, methodological tendencies, and recurring patterns of cognitive barriers reported in previous studies on students' understanding of integral concepts, particularly in relation to graphical and symbolic representations.

The review procedure was guided by the Preferred Reporting Items for Systematic Reviews and Meta-Analyses or PRISMA 2020 framework. PRISMA was used to ensure transparency in reporting the identification, screening, eligibility assessment, and inclusion of studies. The PRISMA flow diagram in this study was revised to follow the PRISMA 2020 format and to present a consistent account of the number of records identified, screened, assessed for eligibility, and included in the final synthesis. The use of PRISMA also allowed the review process to be traced and evaluated systematically by readers.

Data Sources and Search Strategy

The literature search was conducted using four academic databases: Google Scholar, Garuda (Garba Rujukan Digital), ERIC (Education Resources Information Center), and ScienceDirect. These databases were selected because they provide access to international and national publications in mathematics education, calculus learning, mathematical representation, and students' conceptual understanding.

The search covered studies published between 2016 and 2025. The document types considered for inclusion were empirical journal articles and peer-reviewed conference proceedings. Theses, dissertations, books, book chapters, editorials, opinion papers, practice reports without empirical data, and non-academic documents were excluded from the final synthesis. Publications were limited to studies written in English or Indonesian, as these languages were relevant to the databases used and could be evaluated directly by the reviewer.

The search was conducted on 2025. The search strategy combined keywords related to integral concepts, graphical representation, symbolic representation, student difficulties, misconceptions, and cognitive barriers. Boolean operators such as AND and OR were used to refine the search results. The search strings were adjusted according to the search features of each database.

Table 1. Search Strategy Used in Each Database

Database	Search string	Search limitation
Google Scholar	– "integral" AND ("graphical representation" OR "symbolic representation") AND ("student difficulties" OR misconception OR "cognitive barriers")	– 2016–2025; English/Indonesian; journal articles and proceedings; theses and dissertations excluded
Google Scholar	– "definite integral" AND "student difficulties" AND representation	– 2016–2025; empirical studies; only relevant academic articles retained
Garuda	– integral AND representasi AND kesulitan mahasiswa	– 2016–2025; Indonesian-language journal articles and proceedings
Garuda	– miskonsepsi integral AND representasi grafis simbolik	– 2016–2025; empirical articles
ERIC	– ("definite integral" OR "integral calculus") AND ("student difficulties" OR misconceptions) AND ("graphical representation" OR "symbolic representation")	– 2016–2025; peer-reviewed journal articles and proceedings; English
ScienceDirect	– ("integral calculus" OR "definite integral") AND ("graphical representation" OR "symbolic representation") AND ("student understanding" OR misconception OR difficulty)	– 2016–2025; research articles and conference papers; English

Google Scholar was used as a complementary database because it indexes a broad range of academic and institutional documents. However, because Google Scholar produces a large and heterogeneous set of results, the search was limited to the most relevant results based on the ranking provided by Google Scholar. The first 100 most relevant records from each Google Scholar search string were screened. Records from Google Scholar were further filtered manually by examining the title, abstract, source type, publication year, and relevance to the focus of this review. Theses, dissertations, institutional reports, and non-empirical documents identified through Google Scholar were excluded from the final synthesis.

All records retrieved from the four databases were compiled in a data extraction sheet. Duplicate records were checked manually based on title, author name, year of publication, and publication source. Duplicate records, if found, were removed before title and abstract screening. If no duplicate record was identified, this was noted explicitly in the PRISMA flow diagram as "No duplicate records were identified" rather than being reported as "Records excluded, n = 0," to avoid confusion in the identification stage.

Inclusion and Exclusion Criteria

The inclusion and exclusion criteria were established to ensure that the selected studies were directly relevant to the focus of the review. The main focus of this review was empirical evidence concerning students' cognitive barriers in moving from graphical to symbolic representations of integral concepts.

Table 2. Inclusion and Exclusion Criteria

Aspect	Inclusion criteria	Exclusion criteria
Publication year	Studies published between 2016 and 2025	Studies published before 2016
Document type	Empirical journal articles and peer-reviewed conference proceedings.	Theses, dissertations, books, book chapters, editorials, opinion papers, practice reports without empirical data.
Research participants	Undergraduate students or students in higher education contexts.	Elementary, junior high school, or senior high school students
Mathematical topic	Integral concepts, especially definite integrals, area under or between curves, integral limits, integrand functions, and Riemann sums.	Studies focusing mainly on derivatives, limits, algebra, geometry, statistics, or other topics outside integral concepts.
Representation focus	Studies discussing graphical representation, symbolic representation, or the transition between graphical and symbolic representations.	Studies that do not address mathematical representation.
Type of finding	Studies reporting empirical findings on students' difficulties, misconceptions, reasoning processes, or cognitive barriers.	Studies without empirical data or without extractable findings related to students' barriers.
Language	English or Indonesian.	Languages other than English or Indonesian.

The eligibility boundaries were made consistent throughout the review. Therefore, only empirical journal articles and peer-reviewed conference proceedings were included in the synthesis. Theses and dissertations were not included as primary studies, although they could be used only as supplementary background sources if needed and were not counted as part of the final included studies.

Study Selection Procedure

The study selection process followed the PRISMA 2020 stages: identification, screening, eligibility assessment, and inclusion. The initial search across Google Scholar, Garuda, ERIC, and ScienceDirect identified 106 records. These records were checked for duplication and relevance before proceeding to the screening stage.

During the screening stage, titles and abstracts were examined to determine whether the records were related to students, integral concepts, and graphical or symbolic representations. A total of 56 records were excluded at this stage because they were not relevant to the research focus, did not involve higher education students, focused on mathematical topics other than integrals, or did not constitute empirical academic publications. This process resulted in 50 full-text articles being assessed for eligibility.

At the eligibility stage, the full texts of the remaining 50 articles were reviewed. The review focused on whether each article explicitly discussed integral concepts, involved students as participants, reported empirical findings, and provided evidence related to graphical and/or symbolic representations. From this process, 15 articles were found to meet the initial eligibility criteria.

However, after a further appraisal of methodological quality and relevance to the specific focus of this review, 4 of the 15 eligible articles were excluded from the final synthesis. These four articles were removed because they met the broad eligibility criteria but did not provide sufficiently specific evidence for the synthesis of cognitive barriers in the graphical-symbolic transition. More specifically, the reasons for exclusion included one or more of the following: insufficient methodological detail, limited explanation of instruments, lack of extractable findings related to graphical-symbolic transition, or a primary focus on general instructional intervention rather than students' cognitive barriers in integral representation. Therefore, 11 studies were finally included in the systematic review.

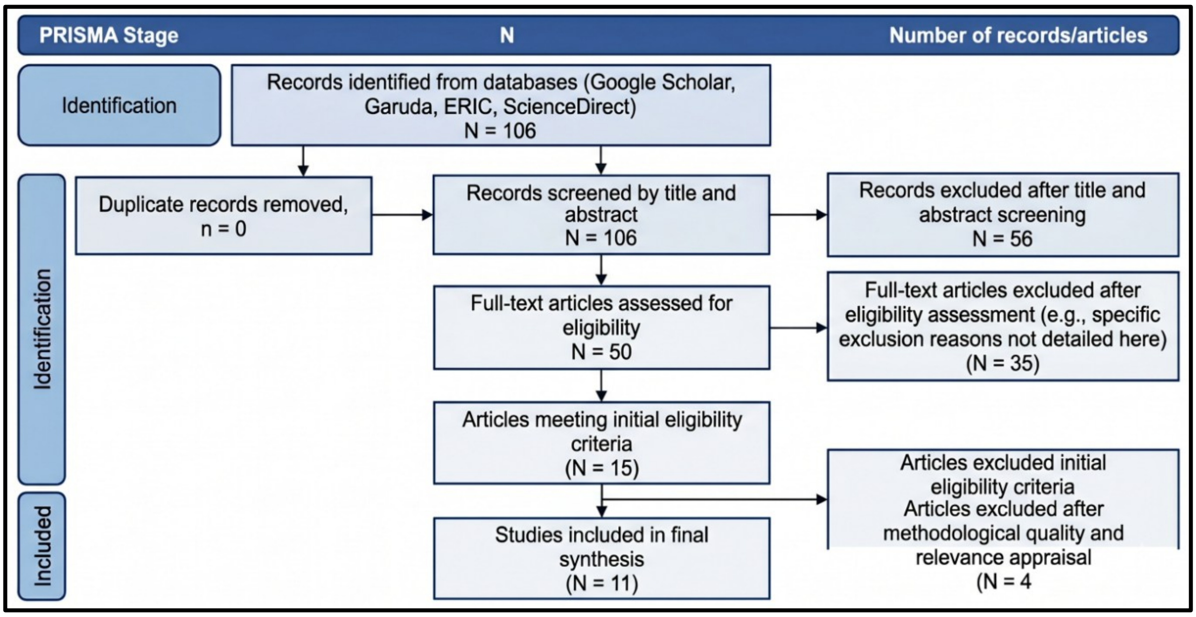

Figure 1. Summary of Study Selection Based on PRISMA 2020

For the PRISMA diagram, the phrase "Records excluded, n = 0" should be removed from the identification stage because it may confuse readers. If no duplicates were found, the diagram should state "Duplicate records removed, n = 0" or "No duplicate records were identified." If duplicates were found, the exact number of duplicates should be reported.

Methodological Quality Appraisal

A methodological quality appraisal was conducted to assess the credibility, clarity, and relevance of the studies included in the final synthesis. The purpose of this appraisal was not to compare studies statistically but to evaluate whether the evidence used in the synthesis was methodologically adequate and relevant to the research question.

Each included study was assessed based on six aspects: study design, participants, research instruments, data analysis methods, relevance to graphical-symbolic transition, and methodological strengths or limitations. These aspects were selected because they directly affect the reliability of the evidence concerning students' cognitive barriers in integral representation.

The quality of each study was evaluated descriptively using three categories: high, moderate, and low. A study was categorized as high quality if it clearly described its research design, participants, instruments, analytical procedures, and provided findings that were directly relevant to graphical-symbolic transition in integral concepts. A study was categorized as moderate quality if it met most of these criteria but had some limitations, such as incomplete reporting of instruments or analytical procedures. A study was categorized as low quality if its methodological information was limited or its relevance to the review focus was weak.

Table 3. Methodological Quality Appraisal

Study	Design	Participants	Instrument / data source	Relevance and limitation	Appraisal
Romero Osorio et al. (2025)	Qualitative / APOS analysis	University students	Integral tasks/interviews	High relevance to conceptual construction and encapsulation.	High
Garcia-Garcia et al. (2025)	Qualitative	Pre-university / transition learners	Derivative and integral connection tasks	High relevance to representation connections and alternative conceptions.	High
Uripno et al. (2024)	Qualitative error analysis	University students	Integral problem solving and AI-supported reflection	Relevant to limit and integrand identification errors.	Moderate
Stevens & Jones (2023)	Qualitative / learning study	Undergraduate students	Adding-up-pieces tasks	Relevant to accumulation and graphical-symbolic linkage.	High
Chitiyo / Godfrey (2022)	Case study	Prospective teacher college students	Integral error tasks	Relevant to misconceptions in integral calculus.	Moderate
Greefrath et al. (2021)	Instrument development / empirical testing	University-level learners	Mental model test instrument	Relevant to area and accumulation models.	High
Singleton et al. (2020)	Classroom practice / intervention report	Engineering or calculus students	Hands-on integral activities	Relevant pedagogical evidence, but limited direct barrier coding.	Moderate
Ramdani et al. (2019)	Qualitative error analysis	University students	Mathematical competency indicators	Relevant to representation and reasoning errors.	Moderate
Palha & Spandaw (2019)	Design research / learning sequence	Students in calculus learning context	Accumulation-function learning sequence	Relevant to instructional design for accumulation.	Moderate
Tatar & Zengin (2016)	Experimental / technology-supported learning	University students	GeoGebra-based definite-integral tasks	Relevant to dynamic visualization and conceptual understanding.	High
Radmehr (2016)	Qualitative / taxonomy-based analysis	University calculus learners	Integral calculus tasks analyzed with revised Bloom taxonomy	Relevant to higher-order reasoning, though thesis-based source needs publication verification.	Limited-to-Moderate

The quality appraisal table was used to support the interpretation of the synthesis. Studies with stronger methodological clarity were given greater interpretive weight in identifying recurring barriers, while studies with methodological limitations were used more cautiously. This procedure helped ensure that the synthesis did not merely summarize findings but also considered the strength of the evidence underlying those findings.

Reviewers, Screening, and Coding Process

The screening and coding process was conducted by reviewer. The procedure should be reported according to the actual process used in the study. The screening of titles, abstracts, and full texts was conducted independently by two reviewers. Each reviewer applied the inclusion and exclusion criteria to determine whether each record should be excluded or retained. Disagreements between reviewers were resolved through discussion until consensus was reached. Inter-rater agreement was calculated using percentage agreement, resulting in an agreement level. This process was used to improve the reliability and transparency of the study selection and coding procedures.

Data Extraction

Data from the included studies were extracted using a structured extraction sheet. The extracted information included author(s), year of publication, research context, study design, participants, integral concept discussed, type of representation examined, research instruments, data analysis methods, main findings, reported student difficulties, and relevance to the graphical-symbolic transition.

The data extraction process focused particularly on findings related to students' difficulties in interpreting graphs, determining integral limits, understanding signed area, connecting integrand functions with graphical regions, translating visual information into symbolic notation, and decomposing regions involving more than one curve.

Table 4. Data Extraction

Author (Year)	Title	Research Focus	Method	Relevant Key Findings	Theoretical Framework
LA Romero Osorio, E Aldana Bermúdez… (2025)	Encapsulation of the definite integral mathematical object of university students: a view from APOS theory	Students' understanding of the concept of definite integrals	Qualitative	Students experience difficulties in the process of encapsulating processes into objects, which is related to the transition from graphical (broad) representation to formal definition.	APOS Theory
J García-García, CA Rodríguez-Nieto… (2025)	Alternative conceptions emerging in pre-university students while making mathematical connections in derivative and integral tasks	Mathematical connections and misconceptions in integrals	Qualitative	Pre-university students have misconceptions in connecting graphical and symbolic representations, especially in the interpretation of area.	Mathematical Connections
G Uripno, S Suprihatiningsih… (2024)	Artificial Intelligence Integration: Error Self-Reflection in Solving Integral Problems	Analysis of errors in solving integrals	Qualitative	Students make mistakes in identifying the limits of integration and the integrand function, which indicates weak translation from the	Error Analysis

Author (Year)	Title	Research Focus	Method	Relevant Key Findings	Theoretical Framework
				problem (graphical/verbal) to symbolic notation.	
BN Stevens, SR Jones (2023)	Learning integrals based on adding up pieces across a unit on integration	Integral understanding as accumulation	Qualitative	The "adding up pieces" approach helps students understand the relationship between graphical (partition) and symbolic (Riemann sum) representations.	Accumulation Theory
G Chitiyo / C Godfrey (2022)	Errors and misconceptions in integral calculus: a case of primary teachers' college students in Zimbabwe	Errors and misconceptions in integrals	Case study	Prospective teacher students make conceptual errors in translating from graphs to symbols, especially in determining the limits of integration and interpreting the area under the curve.	Misconception
G Greefrath, R Oldenburg, HS Siller, V Ulm… (2021)	Basic mental models of integrals: theoretical conception, development of a test instrument, and first results	Basic mental model of integrals	Instrument Development	Identifying students' mental models of integrals, including the "area" and "accumulation" models, which influence graphical-symbolic translation abilities.	Basic Mental Models
L Singleton, E Davishahl, T Haskell (2020)	Getting your hands dirty in integral calculus	Integral learning based on hands-on activities	Classroom Practice	The hands-on (physical) approach helps students build an understanding of the relationship between physical areas and integral symbolic representations.	Contextual Learning
Y Ramdani, N Kurniati, E Harahap… (2019)	Analysis of student errors in integral concepts based on the indicator of mathematical competency using orthon classification	Error analysis based on mathematical competence	Qualitative	Students' errors in integral concepts are related to weak representation and reasoning competencies, especially in graph-based questions.	Orthon Classification
S Palha, J Spandaw (2019)	The Integral as Accumulation Function Approach: A Proposal of a Learning Sequence for Collaborative Reasoning.	Integral approach as an accumulation function	Design Research	Proposes a learning sequence that emphasizes integral understanding as an accumulation to bridge the gap between intuitive (graphical) and formal (symbolic) understanding.	Accumulation Function
E Tatar, Y Zengin (2016)	Conceptual understanding of	Integral conceptual	Experiment	The use of Geogebra helps students visualize the relationship	Conceptual Understanding

Author (Year)	Title	Research Focus	Method	Relevant Key Findings	Theoretical Framework
	definite integral with Geogebra	understanding with Geogebra		between graphical and symbolic representations, thereby reducing misconceptions.	
F Radmehr (2016)	Exploring students' learning of integral calculus using revised Bloom's taxonomy	Exploration of integral calculus learning	Qualitative	Students tend to be at a low cognitive level (remembering, understanding) and have difficulty at the application and analysis levels which require translation of representations.	Revised Bloom's Taxonomy

The extraction sheet was used to ensure that the synthesis was based on comparable information across studies. It also helped trace how each theme was derived from the empirical findings reported in the included studies.

Data Analysis and Thematic Coding

The data analysis combined descriptive synthesis and thematic coding. Descriptive synthesis was used to summarize the characteristics of the included studies, such as publication year, study design, participants, instruments, and data analysis methods. Thematic coding was used to identify recurring patterns of students' cognitive barriers in the transition from graphical to symbolic representations of integral concepts.

The coding procedure consisted of five stages. First, each included article was read in full to understand its research context, objectives, methodology, and findings. Second, relevant segments from the results and discussion sections were extracted, especially those describing students' difficulties in interpreting graphs, determining integral boundaries, understanding area under or between curves, identifying integrand functions, and translating graphical information into symbolic notation. Third, the extracted findings were assigned initial codes. Fourth, similar codes were compared and grouped into broader categories. Fifth, the categories were synthesized into final themes that represented recurring cognitive barriers across the included studies.

The development of themes was both inductive and deductive. It was inductive because the themes emerged from repeated patterns found in the reviewed studies. It was deductive because the coding process was guided by the conceptual focus of this review, namely the transition between graphical and symbolic representations in integral learning. Through this process, three main themes were identified: Visual Attribute Disorientation**,** Cognitive Conflict in Signed Area, and Multicurve Decomposition Barriers.

Table 5. Example of Thematic Coding Process

Final theme	Extracted data example	Initial code	Explanation of theme
Visual Attribute Disorientation	Students misinterpreted the relationship between the graph, the axis, the interval, and the region to be integrated.	Misreading graphical attributes; incorrect integral limits; failure to connect graph and symbolic notation.	This theme refers to students' disorientation in identifying relevant visual attributes, such as curves, axes, intersection points, intervals, and shaded regions.
Cognitive Conflict in Signed Area	Students understood definite integrals as area but experienced difficulty	Area always considered positive; sign error; conflict	This theme reflects the cognitive conflict between the geometric

	when the region was below the x-axis or when the integral value was negative.	between geometric area and signed integral value.	interpretation of area and the symbolic meaning of definite integral as signed area.
Multicurve Decomposition Barriers	Students had difficulty separating regions bounded by more than one curve or determining which function should be placed as the upper or lower function.	Failure to decompose regions; incorrect upper-lower function identification; incorrect construction of multiple integrals.	This theme refers to students' difficulty in decomposing complex graphical regions into appropriate symbolic integral expressions.

To maintain consistency in coding, a codebook was developed. The codebook contained the code name, operational definition, indicators, examples of extracted findings, and the related final theme. The codebook served as a guide for organizing and interpreting findings from the included studies.

Table 6. Codebook for Thematic Analysis

Code	**Operational definition**	**Indicators**	**Example of extracted finding**	**Related theme**
Misreading graphical attributes	Students incorrectly identify or interpret visual elements of a graph that are relevant to integral construction.	Incorrectly reading intersection points, intervals, axes, curves, or shaded regions.	Students determine integral limits based only on the visible shape of the graph rather than the actual intersection points.	Visual Attribute Disorientation
Integral limit error	Students fail to determine the correct lower and upper limits of integration from a graphical representation.	Incorrect interval selection; reversed limits; ignoring relevant boundaries.	Students write an integral over an interval that does not match the shaded region.	Visual Attribute Disorientation
Symbol-graph mismatch	Students cannot connect graphical information with symbolic integral notation.	Incorrect integrand; mismatch between graph and formula; inability to translate visual data into symbols.	Students identify the shaded area correctly but write an unrelated symbolic expression.	Visual Attribute Disorientation
Signed-area misconception	Students misunderstand the meaning of signed area in definite integrals.	Treating all areas as positive; ignoring negative regions; adding areas without considering sign.	Students calculate the area below the x-axis as positive without interpreting its signed contribution.	Cognitive Conflict in Signed Area
Geometric-symbolic conflict	Students experience conflict between the geometric meaning of area and the symbolic value of definite integral.	Confusion between total area and net area; inability to distinguish area from integral value.	Students believe that a definite integral cannot be negative because area is always positive.	Cognitive Conflict in Signed Area
Upper-lower function error	Students incorrectly determine which function is above or below in a region bounded by two curves.	Reversed integrand order; incorrect subtraction of functions.	Students write lower function minus upper function when constructing the integral.	Multicurve Decomposition Barriers

Code	Operational definition	Indicators	Example of extracted finding	Related theme
Region decomposition error	Students fail to divide a complex region into appropriate subregions before constructing integral expressions.	Using one integral for a region requiring multiple intervals; ignoring curve intersections.	Students use a single integral even though the upper and lower functions change across intervals.	Multicurve Decomposition Barriers
Multicurve boundary confusion	Students misidentify boundaries in regions involving more than one curve.	Confusion between curves; incorrect intersection points; wrong interval division.	Students cannot determine the correct boundaries of a region enclosed by multiple curves.	Multicurve Decomposition Barriers

Synthesis of Findings

The synthesis was conducted by comparing the coded findings across the 11 included studies. Findings with similar meanings were grouped to identify recurring cognitive barriers. The synthesis emphasized conceptual patterns rather than statistical aggregation because the included studies varied in terms of research design, participants, instruments, and analytical methods.

No meta-analysis was conducted because the studies did not report sufficiently homogeneous quantitative data for statistical pooling. Instead, quantitative information reported in the primary studies, such as percentages of errors, test scores, or frequencies of misconceptions, was extracted and presented descriptively when relevant. Statistical procedures reported in the primary studies, such as t-tests, ANOVA, correlation analysis, or percentage analysis, were recorded as reported by the original authors. No reanalysis of raw data was conducted in this review.

The final synthesis was organized around the three main themes generated through thematic coding: Visual Attribute Disorientation, Cognitive Conflict in Signed Area, and Multicurve Decomposition Barriers. These themes were used to explain how students experience cognitive barriers when interpreting graphical representations and transforming them into symbolic integral expressions.

Methodological Limitations

Several methodological limitations should be acknowledged. First, the review was limited to studies published between 2016 and 2025 and written in English or Indonesian; therefore, relevant studies published outside this period or in other languages may not have been included. Second, Google Scholar was used as a complementary database, but its broad indexing system may have produced heterogeneous results. To reduce this limitation, Google Scholar results were screened manually and limited to the most relevant academic publications.

Third, the final synthesis included only journal articles and peer-reviewed conference proceedings. Theses and dissertations were excluded to maintain consistency with the inclusion criteria. Fourth, if the screening and coding process was conducted by only one reviewer, the possibility of reviewer subjectivity should be acknowledged because inter-rater agreement could not be calculated. Finally, the review did not conduct meta-analysis because the included studies differed in research design, instruments, and reporting formats. As a result, the synthesis was presented descriptively and thematically.

RESULTS AND DISCUSSION

Results

The analysis of the eleven selected studies yielded three main categories of findings that align with the research questions: (1) the distribution of the literature and theoretical trends underlying the study of cognitive barriers, (2) a typology of cognitive barriers in the transition from graphical to symbolic representation of integral concepts, and (3) factors explaining why this transition is difficult for students. A table is used to summarize the distribution of theoretical characteristics and findings of barriers reported in the articles.

Table 7. Summary of Research on the Transition of Symbolic Representation on the Integral Concept

Study	Country/ context	Participant level	Sample size	Integral task type	Transition studied	Main barrier identified
Romero Osorio et al. (2025)	Colombia / university context	University students	428 students	Definite-integral conceptual tasks	Graphical/ area to formal object	Visual attribute disorientation; encapsulation difficulty
Garcia-Garcia et al. (2025)	Latin American pre-university context	Pre-university students	25 students	Derivative-integral connection tasks	Graphical, algebraic, and symbolic connections	Signed-area conflict; multicurve interpretation
Uripno et al. (2024)	Indonesia / university context	University students	10 students	Integral problem-solving reflection	Problem/graph /verbal to symbolic notation	Limit and integrand identification error
Stevens & Jones (2023)	Undergraduate mathematics education context	Undergraduate students	11 – 12 Partisipant	Adding-up-pieces / Riemann sum tasks	Graphical partition to symbolic accumulation	Accumulation coordination
Chitiyo / Godfrey (2022)	Zimbabwe / teacher college context	Prospective teachers	64 prospective teachers	Integral error and misconception tasks	Graphical to symbolic interpretation	Limit, area, and signed-area misconceptions
Greefrath et al. (2021)	European / German-speaking mathematics education context	Students / learners in calculus context	200 – 300 students	Mental-model test instrument	Area model and accumulation model	Conflict between area and accumulation
Singleton et al. (2020)	Engineering calculus classroom context	Calculus students	24 engineering students	Hands-on physical modeling activity	Physical/ graphical model to symbolic integral	Pedagogical bridging evidence
Ramdani et al. (2019)	Indonesia / university context	University students	45 students	Integral error analysis tasks	Graph-based problems to symbolic reasoning	Representation and reasoning errors
Palha & Spandaw (2019)	Design-research learning context	Calculus learners	2 classes (58 students total)	Accumulation-function learning sequence	Intuitive accumulation to formal symbolism	Accumulation-based transition
Tatar & Zengin (2016)	Turkey / GeoGebra learning context	University students	45 students	GeoGebra definite-integral tasks	Dynamic graph to integral notation	Visualization-supported conceptual understanding
Radmehr (2016)	New Zealand / university context	University calculus learners	117 Students manuscript	Integral calculus tasks using Bloom taxonomy	Conceptual and analytical representation tasks	Higher-order analysis and decomposition difficulty

Literature distribution and theoretical trends

The eleven included studies were published between 2016 and 2025 and represent a focused body of research on integral understanding, representation, accumulation, and student misconceptions. The studies involve university students, prospective teachers, undergraduate learners, and learners transitioning into advanced calculus contexts. The theoretical distribution shows that no single theory fully explains all reported difficulties. Instead, the literature uses complementary frameworks: Duval's theory explains conversion across registers; APOS theory explains the construction and encapsulation of the definite integral as an object; mental model theory explains the tension between area and accumulation images; and accumulation perspectives explain how integral notation can be grounded in summing quantities.

Table 8. Theoretical frameworks and their interpretive roles

Framework	Main sources	Role in this review	Clarification
Semiotic representation / register theory	Duval; representation-focused studies	Explains difficulty in converting graphical information into symbolic notation.	Used explicitly in some studies and interpretively in this review.
APOS theory	Romero Osorio et al. (2025)	Explains action-process-object-schema development and encapsulation of definite integral.	Direct theoretical framework in the study.
Mental models	Greefrath et al. (2021)	Explains area model versus accumulation model and related conflicts.	Direct framework for interpreting signed-area difficulty.
Accumulation theory / accumulation function	Stevens & Jones (2023); Palha & Spandaw (2019)	Explains integral as adding up quantities and supports graphical-symbolic bridging.	Instructional and conceptual framework.
Error analysis / misconception framework	Uripno et al. (2024); Chitiyo / Godfrey (2022); Ramdani et al. (2019)	Identifies recurring student errors in limits, integrands, and regions.	Useful for empirical classification of barriers.
Cognitive taxonomy	Radmehr (2016)	Explains why decomposition and symbolic construction require higher-order analysis.	Relevant to advanced mathematical thinking.

Typology of cognitive barriers in graphical-to-symbolic transition

The thematic synthesis identified three central barriers. These barriers are not merely isolated error types; they represent different points at which students lose mathematical meaning during conversion from graph to symbol.

Table 6. Typology of cognitive barriers with examples of student errors

Barrier	Definition	Concrete error example	Studies reporting	Theoretical interpretation
Visual attribute disorientation	Students select visually salient but mathematically irrelevant graph features, such as y-intercepts, as limits of integration.	Writing limits from y-values instead of projecting region boundaries to the x-axis.	Romero Osorio et al. (2025); Garcia-Garcia et al. (2025); Uripno et al. (2024); Chitiyo/Godfrey (2022); Ramdani et al. (2019).	Duval: failed register conversion; APOS: incomplete object construction.
Cognitive conflict in signed area	Students interpret area visually as always positive and fail to connect it with the signed accumulation represented by definite integrals.	Treating the integral of a region below the x-axis as positive without considering negative function values.	Garcia-Garcia et al. (2025); Chitiyo/Godfrey (2022); Greefrath et al. (2021); Ramdani et al. (2019).	Mental models: area model conflicts with accumulation model.
Multicurve decomposition barriers	Students struggle to split complex regions, identify intersections, and determine which	Using a single top-minus-bottom formula even when curves intersect and	Garcia-Garcia et al. (2025); Uripno et al. (2024); Ramdani et	Advanced reasoning: requires analysis, spatial reasoning, and

Barrier	Definition	Concrete error example	Studies reporting	Theoretical interpretation
	function is upper or lower across intervals.	the upper function changes.	al. (2019); Radmehr (2016).	algebraic validation.

Barrier 1: Visual attribute disorientation

Visual attribute disorientation occurs when students attend to the most visually salient part of a graph rather than the mathematically relevant feature needed for symbolic notation. In definite integrals with respect to x, the lower and upper limits should correspond to x-values that define the domain of the region. However, several studies report that students may use y-intercepts or visible height values as limits. This suggests weak coordination between the graphical register and the symbolic role of the independent variable. From Duval's perspective, the error reflects a failure of conversion; from APOS theory, it indicates that the definite integral has not yet been encapsulated as a flexible mathematical object.

Barrier 2: Cognitive conflict in signed area

The second barrier appears when students interpret all shaded areas as positive quantities and do not connect graphical area with signed accumulation. When a graph lies below the x-axis, the definite integral can be negative because it accumulates function values with sign. Students who rely only on an intuitive area image may write or interpret the symbolic integral incorrectly. Greefrath et al. (2021) help explain this issue through the distinction between area and accumulation mental models. Learning that overemphasizes geometric area without discussing signed accumulation can strengthen this conflict.

Barrier 3: Multicurve decomposition barriers

Multicurve decomposition barriers occur in problems involving regions bounded by two or more curves. Students must identify intersection points, determine the relevant interval or subintervals, and decide which function is above or below on each interval. The difficulty increases when functions intersect and the hierarchy changes. This barrier requires simultaneous spatial reasoning and algebraic validation. It therefore corresponds to higher cognitive levels of application and analysis, as described in Bloom-based and advanced mathematical thinking perspectives.

Discussion

The synthesis suggests that graphical-to-symbolic transition in integral learning is a representational coordination problem rather than a simple procedural weakness. Students may know how to evaluate integrals once symbolic notation is provided, but they may fail when they must construct that notation from a graph. This distinction is important because instruction that emphasizes computation alone may not address the cognitive source of the difficulty.

The three barriers appear to form a hierarchy of representational demands. Visual attribute disorientation reflects an initial difficulty in selecting relevant graphical information. Cognitive conflict in signed area reflects a deeper conflict between intuitive visual meaning and formal symbolic convention. Multicurve decomposition barriers require more advanced coordination because students must segment a graphical region, identify changing relationships, and construct one or more symbolic expressions. Thus, the typology proposed in this review differs from general error classifications by locating errors within the specific process of graphical-to-symbolic conversion.

The theoretical frameworks reviewed in this article are complementary. Duval's theory explains why conversion between registers is cognitively demanding. APOS theory explains why students who have not encapsulated the definite integral as a mathematical object may remain tied to procedures or surface features. Mental model theory explains why students can hold competing interpretations of integral as area and integral as accumulation. Accumulation theory provides an instructional bridge by helping students see integral notation as a formal expression of adding up small quantities. Together, these frameworks show that the problem is not only what students calculate but what meaning they preserve while moving from graph to symbol.

The term blind proceduralism is used here as an interpretive synthesis, not as a causal conclusion. It refers to a learning condition in which students apply memorized integral procedures or formulas without adequately interpreting the graphical structure that gives those symbols meaning. Because this review includes eleven studies, the findings should be read as evidence of consistent patterns rather than definitive proof of universal causality.

Barrier-specific pedagogical implications

Table 9. Pedagogical recommendations aligned with each cognitive barrier

Barrier	Recommended instructional response	Implementation focus
Visual attribute disorientation	Projection tasks that require students to mark region boundaries on the x-axis before writing limits.	Students should first identify the independent variable, domain interval, and graphical boundary before using integral notation.
Cognitive conflict in signed area	Signed accumulation tasks using dynamic visualization or GeoGebra to show positive and negative contributions.	Students should compare geometric area, signed area, and net accumulation through multiple examples above and below the x-axis.
Multicurve decomposition barriers	Decomposition activities in which students identify intersections, split intervals, and test upper-lower relationships before writing integrals.	Students should justify each subinterval and function hierarchy verbally, graphically, and symbolically.

Limitations and future research

This review has several limitations. First, the final synthesis included only eleven studies, so the findings should be interpreted as focused patterns rather than exhaustive generalizations. Second, the search was limited to four databases and to studies published between 2016 and 2025; relevant studies outside these databases or years may have been missed. Third, the review used descriptive synthesis rather than meta-analysis because the included studies differed substantially in design, instruments, and reported outcomes. Fourth, formal inter-rater reliability was not calculated for the coding process. Future research should test the proposed barrier typology through empirical classroom studies, develop diagnostic instruments for graphical-to-symbolic transition, and compare the effectiveness of barrier-specific interventions across different calculus contexts.

CONCLUSION

This systematic review suggests that students' cognitive barriers in the graphical-to-symbolic transition of integral concepts are rooted in weak coordination between representational registers. Across the eleven included studies, three recurring barrier types were identified: visual attribute disorientation, cognitive conflict in signed area, and multicurve decomposition barriers. These barriers indicate that students may lose conceptual meaning when they move from visual information to formal notation, particularly when they must identify limits, interpret signed accumulation, or decompose complex regions. The findings indicate that effective integral instruction should not focus solely on computation but should explicitly train students to coordinate graphical and symbolic meanings. Barrier-specific tasks, dynamic visualization, accumulation-based reasoning, and structured error reflection are promising directions for improving students' conceptual understanding of definite integrals.

RECOMMENDATION

Lecturers and curriculum designers are encouraged to integrate graphical-to-symbolic translation tasks into integral learning. Students should be asked not only to calculate integrals but also to construct integral notation from graphs, explain the meaning of limits and integrands, compare area with signed accumulation, and decompose regions bounded by multiple curves. Technology such as GeoGebra can be used to visualize how graphical changes affect symbolic expressions, while reflective error analysis can help students identify and correct misconceptions. Future studies should develop and validate learning tools that target each barrier type and examine their long-term impact on students' representational fluency.

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CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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