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Constructing Advanced Mathematical Thinking on Function Limits through Cognitive Styles and APOS-Based Scaffolding: A Qualitative Case Study

Ayu Aristika, *Wardono, Scolastika Mariani, Walid
Semarang State University, Semarang, Indonesia
*Corresponding Author e-mail: wardono@mail.unnes.ac.id

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Abstract

This study examines patterns of advanced mathematical thinking (AMT) in learning function limits through students' cognitive styles and APOS-based scaffolding. Although previous studies have investigated APOS theory and cognitive styles separately, limited research has explored how both interact in the development of AMT within calculus learning, particularly in the concept of function limits. Adopting a qualitative case study design, data were generated through semi-structured interviews, classroom observations, and document analysis involving 24 mathematics education students at Universitas Islam An Nur Lampung. In this study, cognitive style refers to field-dependent and field-independent learning tendencies, while AMT is represented through indicators of reasoning, abstraction, representation, proof, and problem posing. The analysis employed thematic coding to identify patterns emerging from students' learning experiences during APOS-based scaffolding activities. The findings reveal three interrelated themes. First, students with field-independent cognitive styles demonstrated more flexible and self-directed reasoning when solving limit problems, whereas field-dependent students relied more on guided explanations, visual representations, and collaborative support. Second, the APOS-based scaffolding sequence of action, process, object, and schema illustrated students' gradual movement from procedural manipulation toward conceptual interpretation of limits. Third, collaborative scaffolding activities created opportunities for peer explanation and collective meaning-making that supported mathematical generalization. Across the AMT indicators, reasoning obtained the highest average score (3.6), while proof showed the lowest average score (2.9), indicating that students experienced greater difficulty in constructing formal mathematical justification. These findings indicate patterns showing that students' advanced mathematical thinking emerged through the interaction between individual cognitive tendencies and instructional scaffolding rather than through isolated instructional techniques alone. The study highlights the importance of aligning scaffolding practices with learners' cognitive characteristics in calculus instruction and contributes qualitative insights into the development of AMT in higher mathematics learning.

Keywords: Cognitive style, APOS theory, Scaffolding, Advanced mathematical thinking, Function limits

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INTRODUCTION

In the current era of globalization and rapid digital transformation, Advanced Mathematical Thinking (AMT) has become an essential competency for university students, particularly in mathematics education, where learners are expected to respond critically and creatively to the demands of Industry 4.0. AMT refers to higher-order mathematical cognition involving reasoning, abstraction, representation, proof, and problem posing, which enables students not only to perform procedural calculations but also to construct conceptual understanding, justify mathematical ideas, and solve complex problems in unfamiliar contexts (Adeniji et al., 2023). These competencies are increasingly important in contemporary education because mathematical literacy is no longer limited to symbolic manipulation, but also includes analytical reasoning, reflective thinking, and conceptual flexibility necessary for technological and data-driven environments.

Despite its importance, the development of AMT in higher mathematics learning remains problematic. Existing studies consistently report that university students experience difficulty in transitioning from procedural manipulation to conceptual understanding, particularly in introductory calculus topics such as function limits (Shodikin, 2022; Tall, 2013). However, the unresolved issue in the literature is not merely that students find limits difficult, but rather that the mechanisms through which students develop AMT during limit learning remain insufficiently understood. Most prior studies focus predominantly on students' errors, misconceptions, or achievement outcomes, while fewer studies investigate how higher-order mathematical thinking develops through classroom interaction, instructional support, and individual cognitive differences during the learning process.

Recent international scholarship has highlighted the importance of scaffolding and constructivist approaches in promoting conceptual mathematical understanding. Studies grounded in APOS theory have shown that the Action–Process–Object–Schema sequence supports students' conceptual transitions in calculus, algebra, and advanced mathematical topics by guiding learners from operational activity toward structured conceptual schemas (Gueudet & Winsløw, 2022; Leng et al., 2023; Rahayu et al., 2024). Similarly, research on mathematical scaffolding emphasizes the importance of dialogic interaction, guided questioning, and collaborative meaning-making in facilitating abstraction and reasoning (Bakker et al., 2015). At the same time, studies on cognitive style demonstrate that field-dependent and field-independent learners process mathematical information differently, influencing their problem-solving strategies, metacognitive regulation, and mathematical representation (Mefoh et al., 2021; Anif et al., 2021). Field-dependent students tend to rely more on external guidance and contextual support, whereas field-independent students are generally more autonomous in organizing symbolic relationships and abstract structures.

Although these strands of research provide valuable insights, they have largely developed independently. APOS-based studies generally emphasize instructional sequencing and conceptual construction but often treat learners as cognitively homogeneous. In contrast, cognitive-style studies typically describe differences in mathematical performance or reasoning processes without examining how these differences interact with structured pedagogical frameworks such as APOS. Consequently, there remains limited understanding of how students with different cognitive styles engage with each APOS stage during the development of Advanced Mathematical Thinking in calculus learning. This gap is particularly evident in studies on function limits, where AMT indicators such as abstraction, proof, and mathematical representation are central but rarely examined simultaneously through both cognitive and pedagogical perspectives.

Preliminary observations conducted at Universitas Islam An Nur Lampung further indicate that students frequently struggle to connect symbolic procedures with conceptual reasoning when solving limit problems. Many students can perform algebraic manipulations mechanically but encounter difficulty when required to justify reasoning, interpret graphical representations, formulate mathematical arguments, or generalize conceptual relationships. These observations suggest that students' difficulties may not solely originate from conceptual complexity, but also from differences in how learners process information and respond to instructional support. Such findings reinforce the need for instructional approaches that are responsive to students' cognitive characteristics rather than uniformly implemented across all learners.

To address this issue, the present study employs APOS theory as a conceptual foundation for scaffolding in mathematics learning. Rooted in constructivist learning theory, APOS proposes that mathematical understanding develops progressively through four stages: action, process, object, and schema (Gueudet & Winsløw, 2022). In this study, APOS serves not merely as a teaching sequence, but as an analytical framework for examining how students construct AMT during classroom interaction. The conceptual framework of this study therefore consists of two integrated components. The first component is derived from established theory: APOS stages as the basis of conceptual mathematical development and cognitive style theory as an explanation of differences in information processing. The second component represents the authors' synthesis, namely the proposition that field-dependent and field-independent students may engage differently with each APOS stage, thereby producing distinct patterns of reasoning, abstraction, representation, proof, and problem posing during limit learning.

Accordingly, the novelty of this study does not lie merely in combining APOS theory and cognitive style as separate concepts. Rather, the study contributes a more specific interpretive perspective by qualitatively tracing how cognitive styles interact with each APOS stage in shaping the development of particular AMT indicators during the learning of function limits. The study therefore offers several contributions. Theoretically, it extends the literature on AMT by integrating cognitive psychology and constructivist pedagogical theory into a unified interpretive framework. Methodologically, it contributes a qualitative analysis of students' mathematical thinking through thematic exploration of classroom interaction, interviews, and scaffolding processes. Empirically, it provides evidence regarding differences in how field-dependent and field-independent students engage with APOS-oriented learning activities in calculus classrooms. Contextually, it contributes insights from higher mathematics education in an Indonesian Islamic university setting, which remains underrepresented in international mathematics education research.

Based on these considerations, this qualitative case study aims to explore how students' cognitive styles and APOS-based scaffolding are manifested in the development of Advanced Mathematical Thinking during the learning of function limits. The study does not attempt to measure causal effects or generalize findings to all mathematics classrooms. Instead, it focuses on understanding learning processes, classroom interactions, and meaning construction within a specific educational context. Specifically, this study addresses the following research questions: (1) How do students with different cognitive styles experience and construct understanding of function limits during APOS-based learning activities? (2) How is APOS-based scaffolding enacted in classroom interactions to support students' reasoning, abstraction, and representation in learning limits? and (3) How do students' cognitive styles interact with APOS-based scaffolding in shaping patterns of Advanced Mathematical Thinking throughout the learning process? Through these questions, the study seeks to illuminate how higher-order mathematical thinking develops through the interplay between individual cognitive tendencies and structured instructional support in higher mathematics education.

LITERATURE REVIEW

Advanced Mathematical Thinking

Advanced Mathematical Thinking (AMT) is a higher-order thinking ability in mathematics that involves reasoning, abstraction, generalization, representation, modeling, and formal proof skills (Tall, 2013; Adeniji et al., 2023). AMT serves as an essential foundation for students in understanding complex concepts in calculus, algebra, and mathematical analysis. The main dimensions of AMT include: (1) reasoning, or the ability to connect mathematical ideas logically; (2) abstraction, as the process of forming concepts from concrete objects into more general ideas; (3) problem posing, which is the ability to generate new problems from mathematical situations; (4) proof, or the ability to construct deductive arguments; and (5) representation, which involves the use of symbols, graphs, and models to explain mathematical ideas (Shodikin, 2022). AMT is not only related to cognitive achievement but is also influenced by pedagogical strategies that can facilitate the process of conceptual construction.

Cognitive Style

Cognitive style refers to the way individuals process, organize, and use information in solving problems. In the context of mathematics learning, cognitive style is often studied through the field-dependent and field-independent model, which differentiates individuals based on their level of dependence on context or independence in analytical thinking (Mefoh et al., 2021). Students with a field-independent style tend to find it easier to analyze the structure of mathematical problems, while field-dependent students benefit more from guidance and scaffolding. In addition, Kolb (1984) proposed a model of learning styles based on the experiential learning cycle, which includes convergent, divergent, assimilative, and accommodative types. Its relevance to mathematical problem-solving indicates that cognitive style influences students' ability to perform abstraction and generalization of mathematical concepts (Adeniji et al., 2023).

APOS Theory and Scaffolding

The APOS theory (Action, Process, Object, Schema) was introduced by Dubinsky and colleagues, emphasizing that mathematical understanding develops through four stages: action (manipulation of concrete symbols), process (internalization of actions), object (understanding of a concept as an entity), and schema (integration of objects into a broader conceptual structure). In the context of calculus learning, APOS-based scaffolding can help students progress from manipulative activities to conceptual abstraction through systematic instructional support (Gueudet & Winsløw, 2022). For example, in the topic of limit functions, students can be guided from simple numerical calculations (action), to pattern generalization (process), and finally to understanding limits as abstract concepts (object) that are integrated into the calculus schema.

Learning the Concept of Function Limits

The concept of function limits is among the most challenging topics for students to understand, primarily due to its abstract nature and the involvement of symbolic representation and the concept of infinity. Shodikin (2022) found that students often struggle to connect graphical, numerical, and symbolic representations when learning about limits. The APOS approach has proven effective in enhancing students' conceptual understanding of limits through the stages of action, process, object, and schema, as demonstrated by Oktaviyanthi & Saputro (2020). However, the application of APOS-based scaffolding within the context of students' cognitive styles remains limited, and thus has not yet fully addressed the diverse learning needs present in heterogeneous classroom settings.

Summary of Research Gap and Contributions

Previous studies in mathematics education have produced substantial yet fragmented insights into learning processes from two dominant perspectives. Research grounded in APOS theory has consistently demonstrated its role in supporting conceptual understanding, algebraic and calculus thinking, and problem-solving skills through the structured progression of action, process, object, and schema (Suwanto et al., 2017; Oktaviyanthi & Saputro, 2020; Leng et al., 2023; Rahayu et al., 2024). These studies emphasize how instructional design based on APOS facilitates students' movement from procedural engagement toward conceptual construction. However, learners in these studies are often implicitly treated as cognitively uniform, with limited attention given to how individual differences shape engagement with each APOS stage. At the same time, a separate body of research has examined cognitive style as a key factor influencing mathematical thinking, achievement, and metacognitive regulation. Studies by Agoestanto et al. (2017), Mefoh et al. (2021), Izzatin et al. (2020), and Anif et al. (2021) show that variations such as reflective–impulsive and field-dependent–field-independent styles lead to distinct patterns of reasoning, representation, and problem-solving behavior. Despite offering rich descriptions of students' thinking processes, these studies rarely situate cognitive style within a specific, theory-driven scaffolding framework, particularly in advanced mathematical topics such as function limits. As a result, the two lines of inquiry APOS-based learning and cognitive style research have largely developed in parallel, leaving unanswered questions about how these dimensions interact in authentic classroom contexts.

This separation reveals a significant research gap, especially in relation to the development of *Advanced Mathematical Thinking* (AMT), which inherently involves abstraction, reasoning, representation, and proof. Empirical studies that explicitly explore how cognitive style intersects with APOS-based scaffolding during the learning of conceptually demanding topics like function limits remain limited, particularly within Islamic higher education institutions in Indonesia, where pedagogical practices, institutional culture, and student backgrounds may shape learning experiences in distinctive ways. Responding to this gap, the present qualitative case study focuses on exploring how students with different cognitive styles experience, interpret, and engage with APOS-based scaffolding in the construction of AMT. Rather than evaluating instructional effectiveness in a causal sense, this study seeks to illuminate learning processes, meaning-making, and interaction patterns as they unfold in classroom practice. Theoretically, the study enriches APOS-based learning theory by positioning cognitive style as an interpretive lens for understanding variation in students' engagement across APOS stages. Practically, the findings offer context-sensitive insights for mathematics lecturers at Universitas Islam An Nur Lampung and similar institutions, supporting the design of calculus instruction that is more adaptive, student-centered, and responsive to cognitive diversity, while contributing to a deeper qualitative understanding of how AMT develops in higher mathematics education.

Conceptual Framework & Research Model Diagram

The conceptual framework of this study is grounded in the understanding that Advanced Mathematical Thinking (AMT) develops through the dynamic interaction between individual cognitive characteristics and structured instructional support. Prior research emphasizes that AMT involves higher-order processes such as reasoning, abstraction, representation, and proof, which are essential for engaging with complex mathematical concepts in higher education (Jones, 2020; Burton, 2021; Gueudet & Winsløw, 2022). From a pedagogical perspective, APOS theory provides a well-established framework for supporting the construction of mathematical concepts by guiding learners through successive stages of action, process, object, and schema (Arnon et al., 2020; Dubinsky & Zazkis, 2021; Oktaviyanthi & Saputro, 2020).

However, studies in cognitive psychology and mathematics education have shown that students approach mathematical tasks differently based on their cognitive styles, which influence how they process information, engage with representations, and regulate their reasoning processes (Adeniji et al., 2023; Mefoh et al., 2021; Putri & Nusantara, 2021; Witkin, 2022). Empirical findings further indicate that these cognitive differences play a significant role in shaping students' experiences when learning challenging topics such as function limits, often determining the extent to which scaffolding supports conceptual understanding (Hidayat, 2021; Shodikin, 2022). Building on these insights, the present study conceptualizes APOS-based scaffolding not as a uniform instructional sequence, but as an adaptive pedagogical process that interacts with students' cognitive styles in fostering the qualitative development of AMT in learning function limits.

The diagram in Figure 1 illustrates how students' cognitive styles function as an initial lens through which mathematical tasks are perceived and engaged with during learning. Differences in cognitive style shape students' pacing, depth of reflection, and representational preferences, which in turn influence how they interact with APOS-based scaffolding at each stage of action, process, object, and schema (Adeniji et al., 2023; Mefoh et al., 2021; Putri & Nusantara, 2021; Witkin, 2022). Rather than operating as a linear instructional mechanism, APOS scaffolding dynamically interacts with students' cognitive characteristics through guided prompts, visual mediation, peer dialogue, and reflective questioning, reflecting the constructivist foundations of APOS theory in supporting conceptual development (Arnon et al., 2020; Dubinsky & Zazkis, 2021; Rahmawati et al., 2022).

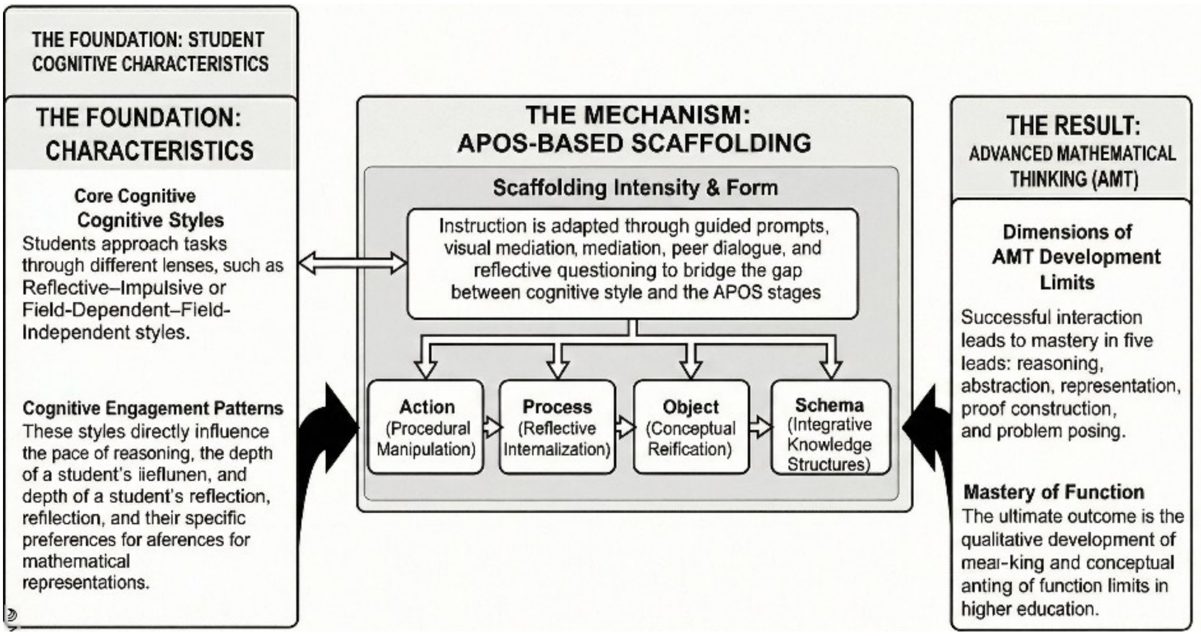

Figure 1. Conceptual Framework

The interaction supports the qualitative development of *Advanced Mathematical Thinking*, manifested in students' reasoning, abstraction, representation, proof construction, and problem posing, which are widely recognized as core dimensions of higher-order mathematical thinking in advanced mathematics learning (Jones, 2020; Burton, 2021; Gueudet & Winsløw, 2022). The diagram emphasizes that AMT emerges as a process of meaning-making shaped by the reciprocal relationship between cognitive style and pedagogical scaffolding, particularly in learning conceptually demanding topics such as function limits, which require sustained abstraction and conceptual coordination across representations (Hidayat, 2021; Shodikin, 2022; Suryadi, 2021).

METHOD

Research Design

This study employed a qualitative case study design to obtain an in-depth understanding of how students' cognitive styles and APOS-based scaffolding were manifested in the development of Advanced Mathematical Thinking (AMT) during the learning of function limits. A qualitative case study was considered appropriate because the study focused on interpreting students' learning experiences, classroom interactions, and meaning-construction processes within a natural instructional setting rather than measuring causal effects or testing statistical relationships among variables. The case was bounded within a Differential Calculus course at Universitas Islam An Nur Lampung, specifically during classroom learning activities related to function limits.

The study adopted an interpretive perspective in which students' reasoning, abstraction, representation, proof construction, and problem-posing activities were examined through classroom interaction, reflective dialogue, and instructional scaffolding aligned with APOS theory. In this context, APOS-based scaffolding functioned as a pedagogical framework embedded within regular instruction rather than as an experimental treatment. The qualitative approach therefore enabled a contextual and process-oriented exploration of how different cognitive tendencies interacted with stages of mathematical understanding during learning activities (Yin, 2020; Baxter & Jack, 2021).

Context and Participants

The study was conducted in the Mathematics Education Program at Universitas Islam An Nur Lampung during the even semester of the 2024/2025 academic year. The research context was a Differential Calculus course in which function limits constituted a major instructional topic.

Participants consisted of 24 mathematics education students from the fourth and sixth semesters who had completed prerequisite introductory calculus content and were actively enrolled during the period of data collection. A purposive sampling strategy was employed to capture variation in mathematical engagement and cognitive characteristics. Participant selection was based on the following criteria:

1. active participation during classroom learning activities;
2. variation in academic achievement levels (high, medium, and low);
3. willingness to participate in interviews and reflective learning activities; and
4. representation of different cognitive style categories identified during the preliminary classification stage.

Classroom observations involved all 24 participants because the study aimed to understand collective classroom interaction during APOS-oriented learning activities. From these participants, 12 students were purposively selected for in-depth semi-structured interviews to provide richer insight into individual reasoning processes and conceptual experiences. The interview participants represented balanced variation across:

1. field-dependent and field-independent cognitive styles;
2. high, medium, and low mathematical achievement; and
3. different levels of classroom participation.

This selection strategy was intended to strengthen the interpretive depth and variation of qualitative findings.

Cognitive Style Identification

Students' cognitive styles were identified using the Group Embedded Figures Test (GEFT) developed by Witkin (2022), which has been widely used in mathematics education research to classify learners into field-dependent (FD) and field-independent (FI) categories. The GEFT was administered prior to the classroom learning activities as part of the preliminary data collection stage. The instrument consisted of visual analytical tasks requiring students to identify simple geometric figures embedded within more complex patterns. Students with higher GEFT scores were categorized as field-independent because they demonstrated stronger analytical differentiation abilities, whereas students with lower scores were categorized as field-dependent due to their greater reliance on external contextual structures.

The classification procedure followed established GEFT scoring guidelines:

1. scores above the institutional mean were categorized as field-independent;
2. scores below the mean were categorized as field-dependent.

To support validity, the instrument was reviewed by two mathematics education experts and piloted with students outside the research participants prior to implementation. The GEFT has demonstrated acceptable reliability and construct validity in previous educational studies examining cognitive processing and mathematical reasoning. The cognitive style classification was subsequently used as an analytical lens for interpreting students' learning experiences, classroom interaction patterns, and AMT development during APOS-based learning activities.

APOS-Based Learning Procedure

The learning activities were organized according to the Action–Process–Object–Schema (APOS) framework. Rather than functioning as a separate instructional intervention, APOS-based scaffolding was embedded within regular classroom learning activities to support students' conceptual understanding of function limits. Learning activities were conducted across four instructional sessions, each lasting approximately 100 minutes. The lecturer facilitated conceptual exploration through guided questioning, reflective prompts, and collaborative discussion, while peer interaction was emphasized particularly during the Process and Object stages.

Table 1. Structure of APOS-Based Learning Activities and Targeted AMT Indicators

Session	APOS Stage	Main Learning Activities	Lecturer's Role	Peer Role	Targeted AMT Indicators
Session 1	Action	Solving basic numerical and algebraic limit problems	Procedural guidance and clarification	Limited peer discussion	Procedural reasoning, representation
Session 2	Process	Identifying patterns and graphical interpretations	Reflective questioning	Strategy comparison in groups	Reasoning, abstraction
Session 3	Object	Interpreting limits conceptually across representations	Conceptual prompting	Peer explanation and justification	Abstraction, proof
Session 4	Schema	Integrating limits with continuity and derivatives	Conceptual synthesis support	Collaborative problem posing	Generalization, problem posing

To maintain consistency of implementation, lesson plans grounded in APOS principles were developed prior to instruction, classroom activities were documented through observation notes, and reflective discussions with the course lecturer were conducted after each session.

Instruments and Data Sources

Data in this study were generated through multiple qualitative instruments and sources in order to obtain a comprehensive understanding of students' learning experiences, classroom interactions, and the development of Advanced Mathematical Thinking (AMT) during APOS-based learning activities. The use of various data sources was intended to support methodological triangulation and to strengthen the credibility of the qualitative interpretation. Semi-structured interviews were conducted with 12 selected students who represented different cognitive styles, levels of mathematical achievement, and patterns of classroom participation. These interviews were designed to explore students' conceptual understanding of function limits, their experiences during APOS-oriented learning activities, their reasoning strategies when solving limit problems, and their perceptions of the scaffolding provided during instruction. Through this interview format, students were given opportunities to explain their thinking processes in detail while allowing the researcher to ask follow-up questions based on participants' responses.

Classroom observation was conducted throughout the APOS-based learning sessions to document how students engaged with instructional activities in a natural classroom setting. The observation focused on lecturer-student interaction, peer discussion, scaffolding practices, and visible manifestations of AMT indicators during the learning process. Particular attention was given to how students responded to guided questioning, visual representations, collaborative explanation, and conceptual prompts across the Action, Process, Object, and Schema stages. Document analysis was also used as an important source of qualitative evidence. The documents analyzed included students' written assignments, reflective learning notes, classroom worksheets, and APOS-oriented learning materials. These documents provided written traces of students' mathematical reasoning, abstraction, representation, proof construction, and problem-posing activities. The analysis of these documents helped confirm and enrich the findings obtained from interviews and classroom observations.

To improve transparency in interpretation, the indicators of Advanced Mathematical Thinking were operationalized through an analytical rubric adapted from previous AMT literature. The rubric was used to guide the interpretation of students' demonstrated abilities in reasoning, abstraction, representation, proof, and problem posing. Rather than functioning as a standardized quantitative test, the rubric served as a qualitative analytical tool for organizing evidence across interviews, observations, and written documents.

Table 2. Operationalization of Advanced Mathematical Thinking (AMT)

AMT Indicator	Operational Definition	Observable Evidence	Coding/Scoring Criteria
Reasoning	Ability to explain logical relationships in solving limits	Verbal explanation, justification of procedures	1–4 scale: fragmented to coherent reasoning
Abstraction	Ability to generalize patterns into conceptual understanding	Transition from numerical examples to general concepts	1–4 scale: concrete to abstract understanding
Representation	Ability to connect symbolic, graphical, and verbal forms	Use of graphs, symbols, verbal interpretation	1–4 scale: isolated to integrated representations
Proof	Ability to construct mathematical justification	Logical argument and formal explanation	1–4 scale: intuitive to structured proof
Problem Posing	Ability to formulate new mathematical problems	Creating related limit problems independently	1–4 scale: simple modification to conceptual generation

The numerical averages reported in the findings section, such as reasoning (3.6) and proof (2.9), were derived from this rubric-based qualitative scoring process. Scores represented interpretive summaries of students' demonstrated AMT performance across observations, interviews, and written work rather than standardized quantitative measurements.

Data Analysis

Data were analyzed using thematic analysis informed by the interactive model of Miles, Huberman, and Saldaña (2019). The analysis combined both theory-driven coding derived from APOS and AMT frameworks and data-driven coding emerging from participants' experiences. The analysis proceeded through several stages:

Data Familiarization

Interview transcripts, observation notes, and student documents were repeatedly reviewed to obtain an overall understanding of participants' experiences.

Open Coding

Initial codes were generated by identifying meaningful segments related to:

1. reasoning,
2. abstraction,
3. representation,
4. proof,
5. problem posing,
6. cognitive style characteristics,
7. and scaffolding interaction.

Axial Coding

Related codes were grouped into broader conceptual categories linking cognitive style, APOS stages, and AMT development patterns.

Theme Refinement

Themes were refined iteratively by comparing evidence across interviews, classroom observations, and written documents to ensure conceptual coherence and representativeness.

Cross-Case Comparison

Comparisons were conducted between field-dependent and field-independent students to identify differences in learning engagement and mathematical thinking patterns.

NVivo 14 software was used to support data organization, coding management, theme categorization, and retrieval of coded excerpts. NVivo facilitated systematic comparison across participants and helped visualize relationships among themes, APOS stages, and AMT indicators through thematic mapping.

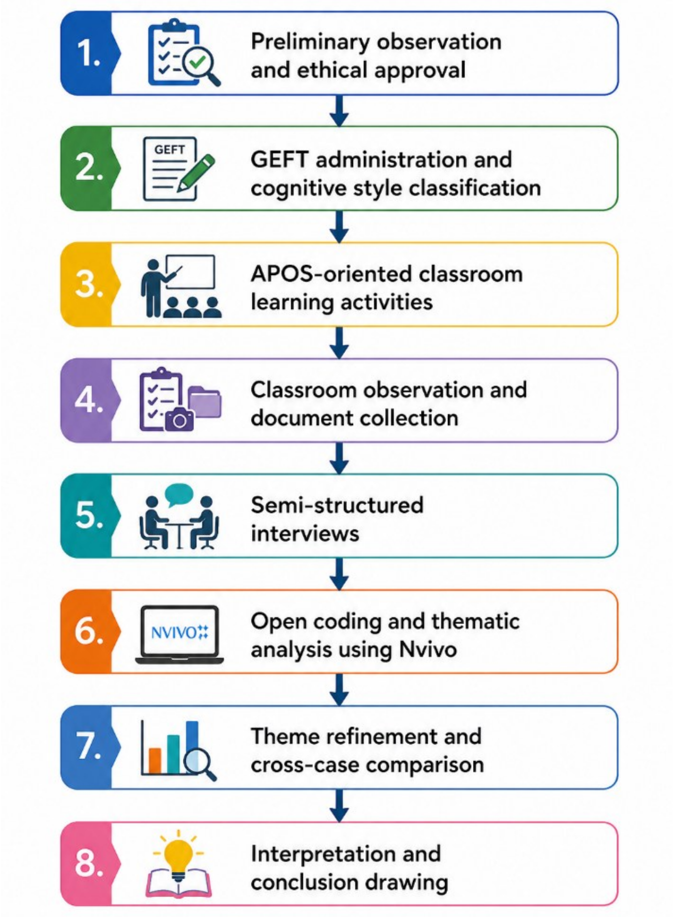

Figure 2. Research Procedure Flowchart

Trustworthiness

Several strategies were employed to strengthen the credibility and trustworthiness of the findings. Since this study used a qualitative case study design, trustworthiness was considered essential to ensure that the interpretations were grounded in sufficient evidence and reflected participants' learning experiences as accurately as possible. The strategies used included triangulation, member checking, peer debriefing, inter-coder checking, and the maintenance of an audit trail throughout the research process.

Triangulation was conducted by comparing data obtained from different sources and methods. Source triangulation involved information collected from students, the lecturer, and learning documents, while method triangulation involved semi-structured interviews, classroom observations, and document analysis. By examining the consistency of findings across these sources and methods, the study was able to develop a more comprehensive and credible interpretation of how students' cognitive styles and APOS-based scaffolding contributed to the development of Advanced Mathematical Thinking.

Member checking was also used to confirm the accuracy of the researcher's interpretations. Preliminary interpretations and thematic summaries were discussed with selected participants to ensure that the meanings constructed from the data were consistent with their actual experiences. This process allowed participants to clarify, confirm, or refine the researcher's understanding of their responses and learning processes.

Peer debriefing was conducted through discussions with two mathematics education researchers who had experience in qualitative inquiry. These discussions focused on reviewing research interpretations, coding decisions, and emerging themes. Peer debriefing helped reduce potential researcher bias and strengthened the analytical rigor of the study by allowing external researchers to question, challenge, and validate the interpretation process.

Inter-coder checking was carried out to ensure the consistency and reliability of thematic categorization. A subset of interview transcripts was independently coded by a second reviewer, and the coding results were compared with the researcher's initial coding. Any differences in coding interpretation were discussed to reach a more coherent and defensible categorization of themes.

An audit trail was maintained throughout the study to enhance transparency and confirmability. This included systematic documentation of data collection procedures, coding decisions, analytical memos, and the process of theme development. The audit trail provided a clear record of how the findings were constructed from the data, making the analytical process more traceable and accountable.

Ethical Considerations

Ethical approval for the study was obtained from the institutional research authority at Universitas Islam An Nur Lampung prior to data collection. Participants were informed about the purpose of the study, the voluntary nature of participation, and their right to withdraw at any stage without academic consequences. Written informed consent was obtained from all participants before interviews and classroom observations were conducted. To protect confidentiality, pseudonyms were used in transcripts and research reporting, and all collected data were stored securely and used solely for academic purposes.

RESULTS AND DISCUSSION

Research Findings

Based on thematic analysis of interview transcripts, classroom observations, student worksheets, and reflective learning documents, three major themes emerged regarding the interaction between cognitive style and APOS-based scaffolding in the development of Advanced Mathematical Thinking (AMT) during the learning of function limits. The findings presented in this section combine qualitative thematic evidence with rubric-based interpretive scoring of AMT indicators. The numerical summaries reported later in the section, such as averages for reasoning and proof, were derived from qualitative rubric assessments on a 1–4 scale evaluating students' demonstrated reasoning, abstraction, representation, proof, and problem-posing abilities across interviews, written work, and classroom interaction.

Theme 1: Variations in Field-Dependent and Field-Independent Cognitive Styles in Understanding the Concept of Limits

Analysis of interview data, classroom interaction, and learning documents showed consistent differences between field-independent (FI) and field-dependent (FD) students in how they approached conceptual understanding of function limits. These differences appeared repeatedly across multiple data sources and therefore represented broader learning patterns rather than isolated individual cases. Students categorized as FI generally demonstrated stronger independence during problem interpretation and symbolic reasoning. They tended to identify patterns autonomously, reorganize information analytically, and move more quickly from procedural activity toward abstraction and conceptual explanation. In contrast, FD students relied more frequently on external clarification, visual examples, lecturer guidance, and peer discussion before constructing conceptual understanding. The following quotation illustrates a recurring pattern among FI students:

"When working on limit problems, I usually try to understand the meaning of the question first on my own before deciding on the steps. I feel more comfortable when I can find the pattern by myself." (Interview, FI-3)

Classroom observations supported this statement. FI students were more likely to attempt symbolic manipulation independently before seeking assistance and frequently initiated alternative strategies during group discussion. By comparison, the following quotation reflects a common pattern observed among FD students:

"I understand more easily when the lecturer explains the steps in detail. If I'm only given problems without examples, I often feel confused about where to start." (Interview, FD-2)

Observation notes indicated that FD students more frequently requested confirmation from lecturers or peers before proceeding with calculations. They also relied more heavily on worked examples and visual demonstrations when interpreting indeterminate forms and graphical representations of limits. A similar pattern emerged during collaborative learning activities

"After group discussion and the explanation was repeated, I finally understood why the limit value cannot be directly substituted. At first, I was just following the formula." (Interview, FD-6)

This quotation reflected a broader tendency among FD students in which conceptual understanding emerged after repeated explanation and collaborative negotiation of meaning rather than through immediate individual abstraction. Rubric-based qualitative scoring also indicated that reasoning demonstrated the highest average development across participants (3.6/4), while proof-related thinking showed the lowest average score (2.9/4). These scores represented interpretive evaluations derived from triangulated qualitative evidence rather than standardized quantitative testing. The findings suggest that students generally developed explanatory reasoning and conceptual representation earlier than formal mathematical proof. The comparison between FI and FD students further indicated that differences in abstraction and proof construction were especially visible during the Object and Schema stages of APOS learning. FI students more frequently interpreted limit operations as conceptual entities and attempted formal justification independently. In contrast, FD students demonstrated stronger engagement during the Action and Process stages, where visual representation, worked examples, and collaborative discussion provided concrete support for conceptual transition.

However, the findings also suggest that cognitive style alone did not fully explain students' mathematical engagement. Several cases indicated that prior mathematical achievement, self-confidence, and familiarity with collaborative learning environments may also have influenced participation and conceptual development. For example, some FD students with stronger academic backgrounds demonstrated abstraction abilities comparable to FI students after extended scaffolding activities, while several FI students showed reluctance in proof-oriented discussion due to low confidence in verbal explanation. Overall, the findings indicate that FI and FD students followed different but meaningful pathways toward understanding function limits. Rather than reflecting fixed differences in mathematical ability, these patterns suggest variation in how students responded to APOS-based scaffolding and engaged with conceptual meaning-making processes during higher mathematics learning.

Theme 2: The Role of APOS-Based Scaffolding in Students' Construction of Function Limit Concepts

Analysis of interview transcripts, classroom observations, reflective notes, and student assignments showed that APOS-based scaffolding supported students' understanding of function limits through learning experiences aligned with the Action, Process, Object, and Schema stages. Across data sources, students consistently described APOS-oriented activities as helping them gradually connect procedures, representations, and conceptual meanings rather than simply memorizing formulas. These patterns appeared repeatedly across participants and therefore reflected broader tendencies rather than isolated classroom incidents. During the Action stage, students frequently relied on worked examples, lecturer prompts, and structured exercises to begin manipulating symbolic expressions. This pattern was particularly visible among field-dependent (FD) students, who often expressed uncertainty when procedural guidance was limited.

"At the beginning, I relied on the worked examples, but when I was asked to explain each step, I started to understand why the procedure works." (Interview, FD-2)

The statement reflects how procedural scaffolding gradually encouraged the student to internalize mathematical relationships rather than merely imitate procedures. Observation notes supported this interpretation, showing that similar FD students initially depended heavily on lecturer modeling but became more active when prompted to explain reasoning verbally during small-group discussion. This quotation therefore represented a broader pattern observed among several FD participants rather than a singular experience. As learning progressed into the Process and Object stages, students increasingly described conceptual reinterpretation of limits beyond procedural calculation.

"After the discussion, I realized that a limit is not just an answer. It is about approaching a value, even if it is never reached." (Interview, FI-3)

The quotation demonstrates movement from operational understanding toward conceptual abstraction, where the student began treating the limit process as a meaningful mathematical object rather than a computational outcome. Classroom observation data indicated that similar responses were more frequently expressed by field-independent (FI) students during reflective questioning activities. These students tended to articulate conceptual explanations independently and were more likely to initiate connections between symbolic manipulation and mathematical meaning. The Schema stage became visible when students connected limits with broader calculus concepts such as continuity and derivatives.

"When we connected limits to graphs and derivatives, I could see the bigger picture. It made the concept more coherent." (Interview, FI-5)

The statement illustrates how students integrated multiple mathematical representations into a broader conceptual structure. Similar patterns appeared in students' written reflections and classroom discussions, where several participants described limits not as isolated topics but as interconnected components of calculus reasoning. The findings indicate that APOS-based scaffolding supported conceptual development through a gradual progression from procedural engagement toward conceptual understanding. However, the learning trajectory was not experienced uniformly across students. FD students generally benefited more from scaffolding emphasizing structured guidance, worked examples, visual representation, and collaborative explanation during the Action and Process stages. In contrast, FI students appeared to engage more productively with reflective prompts, conceptual questioning, and justification tasks during the Object and Schema stages.

This pattern became particularly visible in the development of reasoning and abstraction. FI students more frequently demonstrated independent conceptual restructuring, where symbolic procedures were reinterpreted as conceptual entities. Their explanations often included logical justification and conceptual generalization. Meanwhile, FD students showed stronger development in representation and explanatory reasoning when visual and collaborative scaffolding supported conceptual transition. Rubric-based qualitative scoring further reinforced these patterns. Reasoning obtained the highest average development score across participants (3.6/4), followed by representation and abstraction, while proof-related thinking demonstrated the lowest average score (2.9/4). These scores were not derived from standardized quantitative testing, but from qualitative rubric assessments triangulating interviews, written work, classroom interaction, and observational evidence across AMT indicators.

The findings also suggest that APOS-based scaffolding functioned not as a rigid instructional sequence, but as a flexible pedagogical structure enabling students to revisit and reinterpret understanding across stages. Several students moved recursively between Action, Process, and Object stages when encountering conceptual difficulty, indicating that AMT development occurred through iterative reflection rather than linear progression.

Theme 3: Development of Advanced Mathematical Thinking Indicators Through the Interaction of Cognitive Style and APOS-Based Scaffolding

Analysis of interview data, classroom observations, and learning documents revealed that the development of Advanced Mathematical Thinking (AMT) indicators emerged through interaction between students' cognitive styles and APOS-based scaffolding practices. Indicators such as reasoning, abstraction, representation, and proof developed differently depending on how students engaged with instructional support across APOS stages. Students with field-independent (FI) tendencies generally demonstrated stronger engagement in abstraction, logical explanation, and proof-oriented reasoning, particularly during Object and Schema activities. In contrast, field-dependent (FD) students showed more visible development in representation and procedural reasoning when supported through visual and collaborative scaffolding. The following quotation reflects a recurring pattern among FD students:

"When I used graphs, I could better understand how the value approaches the limit. It helped me explain the idea, even when the calculation was difficult." (Interview, FD-3)

The excerpt indicates that visual scaffolding functioned as a bridge between procedural difficulty and conceptual understanding. Observation data showed that similar FD students relied heavily on graphs and tables to interpret approaching values before engaging confidently with symbolic expressions. The quotation therefore represented a common pattern among FD participants rather than an isolated experience. By comparison, the following statement illustrates a recurring tendency among FI students:

"After understanding the concept, I tried to explain why the limit behaves that way. I felt more confident justifying my answer logically." (Interview, FI-4)

The excerpt demonstrates how FI students tended to move beyond procedural explanation toward logical justification and conceptual coherence. Classroom observations showed that FI students more frequently initiated explanatory arguments during discussion and attempted to connect symbolic operations with broader conceptual relationships. The synthesized findings indicate that AMT indicators developed through differentiated pathways shaped by cognitive style and APOS-oriented scaffolding. The interaction between these two dimensions appeared particularly important in determining how students engaged with reasoning, abstraction, representation, and proof during limit learning. Field-independent students demonstrated stronger abstraction and proof-related development because they appeared more capable of reorganizing symbolic procedures into internally coherent conceptual structures. During the Object and Schema stages, these students frequently interpreted procedures reflectively, evaluated logical consistency, and generalized relationships independently. Their reasoning processes were often recursive and self-regulated, enabling stronger engagement with conceptual proof and mathematical justification.

In contrast, field-dependent students relied more heavily on representation and collaborative explanation as entry points toward higher-order thinking. Graphs, tables, lecturer prompts, and peer discussion appeared to reduce abstraction difficulty by providing observable reference points for conceptual interpretation. Representation therefore functioned not merely as supportive visualization, but as a mediating mechanism through which FD students gradually constructed conceptual meaning before engaging with abstraction and proof. The findings further suggest that reasoning and abstraction tended to emerge earlier than proof-related thinking across both cognitive groups. While many students were able to explain conceptual relationships verbally or graphically, fewer students demonstrated fully structured formal justification. This pattern was reflected in the rubric-based qualitative scoring, where proof obtained the lowest interpretive average among AMT indicators. The scoring process was conducted qualitatively using a 1–4 rubric integrating evidence from written work, interview explanation, classroom participation, and observational notes.

The comparison between FD and FI students also showed that APOS stages interacted differently with each cognitive tendency:

1. FD students showed strongest engagement during Action and Process activities emphasizing representation, guided explanation, and collaborative interpretation.
2. FI students demonstrated stronger conceptual integration during Object and Schema activities emphasizing reflective reasoning and abstraction.

Nevertheless, several cases indicated that cognitive style alone did not fully explain students' AMT development. Prior mathematical achievement, confidence in calculus learning, and familiarity with collaborative discussion also appeared to influence students' participation and conceptual growth. Some FD students with stronger academic preparation demonstrated abstraction abilities approaching those of FI students after sustained scaffolding, while several FI students displayed hesitation in proof-oriented discussion despite strong symbolic reasoning. Overall, the findings indicate that AMT development occurred through reciprocal interaction between cognitive tendencies and instructional mediation rather than through uniform cognitive progression. APOS-based scaffolding therefore functioned as a flexible learning structure that enabled students with different cognitive profiles to access advanced mathematical thinking through distinct but meaningful conceptual pathways.

Documentation Results

Document analysis was conducted to complement interview and observation data and to provide written evidence of students' learning processes and the development of *Advanced Mathematical Thinking* (AMT). The documents analyzed included students' learning reflection notes, limit function task sheets, AMT test results based on an analytic rubric, and lecturer observation notes recorded during APOS-based instruction. These documents were examined to identify patterns of abstraction, reasoning, representation, proof, and conceptual integration, and to verify consistency with the findings reported in Themes 1, 2, and 3.

Table 3. Summary of Documentation Analysis Related to AMT Development

Document Type	Main Findings	Evidence of AMT Development
Learning Reflections	Nine of the analyzed reflections contained conceptual terms such as *approaching value*, *relationship between graphs and symbols*, and *epsilon–delta*.	Indicates the emergence of conceptual awareness and abstraction in understanding limits.
Limit Function Tasks	Approximately 75% of students solved trigonometric limit problems using logical reasoning supported by graphical representations.	Demonstrates development of representation and reasoning abilities.
AMT Test Results (Rubric 0–4)	The overall mean score was 3.28 (high category). Reasoning achieved the highest average (3.6), while proof obtained the lowest average (2.9).	Reflects growth in higher-order thinking skills, with proof developing more gradually.
Lecturer Observation Notes	Students increasingly asked conceptual questions, such as "Why can't the limit value be calculated directly?"	Provides evidence of emerging mathematical curiosity and reflective thinking.

"The APOS stages made me realize that understanding limits cannot be instantaneous. A limit is an idea, not a result. Through group discussions, I feel more confident expressing my opinions." (Reflection, R-6)

The documentation results reinforce the qualitative patterns identified in Themes 1, 2, and 3 by providing written evidence of students' evolving mathematical thinking. Learning reflections reveal a shift from procedural descriptions toward conceptual language, indicating the development of abstraction and meaning-making consistent with APOS-based scaffolding. Task sheets and AMT test results show that reasoning and representation developed more strongly than formal proof, aligning with interview findings that proof especially required sustained reflection and guidance at the *Schema* stage. Lecturer notes further indicate that students began to question underlying concepts rather than merely apply formulas, reflecting the influence of both cognitive style and scaffolding on reflective engagement. Overall, the documentation confirms that the development of AMT occurred through a gradual, supported process in which cognitive tendencies and APOS-based scaffolding jointly shaped students' understanding of function limits.

Triangulation and Thematic Analysis of Advanced Mathematical Thinking (AMT)

Triangulation across multiple data sources was conducted to strengthen the credibility and coherence of the qualitative findings. Observation records, interview transcripts, and learning documentation were examined comparatively to identify convergent patterns related to the development of *Advanced Mathematical Thinking* (AMT). This triangulation confirms that the themes identified in Theme 1 (cognitive style differences), Theme 2 (APOS-based scaffolding processes), and Theme 3 (development of AMT indicators) are consistently reflected across data sources. Rather than serving as isolated evidence, each data source complements the others in explaining how students' reasoning, abstraction, representation, proof, and problem posing evolved through the interaction of cognitive style and APOS-based scaffolding during the learning of function limits.

Table 4. Triangulated Evidence of AMT Development Across Data Sources

AMT Dimension	Indicator	Observation Data	Interview Data	Documentation Data
Reasoning	Connecting symbols with the conceptual meaning of limits	Classroom discussions increasingly involved argumentative explanations, where students justified why certain limit procedures were valid or invalid.	Students articulated the logic behind formulas and solution steps, explaining not only how but why a method was used.	Written responses in essay-type tasks showed coherent logical explanations rather than mere formula substitution.
Abstraction	Drawing conclusions from numerical and graphical patterns	Abstraction became visible during the transition from the *Process* to the *Object* stage, as students shifted focus from calculations to underlying ideas.	"I started to see a pattern from small x values, not just the numbers themselves." *(Interview, Student-4)*	Reflection notes contained generalized statements about limits as approaching values rather than exact results.
Represen-tation	Using symbolic and graphical forms to express concepts	Students frequently drew curves or tables to explain limit behavior during presentations and group discussions.	Students explicitly stated preferences for visual or symbolic representations depending on their cognitive style.	Task sheets and notes included sketches of graphs and coordinated use of symbols and diagrams.
Proof	Constructing formal or semi-formal arguments	Proof-related discussions were still simple, often limited to verbal justification guided by the lecturer.	Students reported increasing confidence in explaining answers logically, although formal proof writing remained challenging.	Rubric-based AMT tests showed improvement in proof scores, though this indicator remained lower than others.

AMT Dimension	Indicator	Observation Data	Interview Data	Documentation Data
Problem Posing	Creating and modifying limit problems	Some students proposed variations of limit problems during group work and class discussion.	Reflections indicated curiosity and emerging ideas for new problems related to limits.	Additional assignments revealed student-generated limit problems with varying degrees of complexity.

The triangulated analysis demonstrates strong consistency across observation, interview, and documentation data, reinforcing the validity of the qualitative findings. Reasoning, abstraction, and representation emerged as the most rapidly developing AMT dimensions, supported by APOS-based scaffolding at the *Action* and *Process* stages and shaped by students' cognitive styles, as identified in Themes 1 and 2. Proof and problem posing showed emerging but slower development, aligning with Theme 3 findings that these indicators require sustained reflection and support at the *Schema* stage. Overall, the triangulation confirms that AMT development occurred through a gradual, interconnected process in which cognitive style influenced how students engaged with scaffolding, while APOS-based instruction provided the structure necessary for higher-order mathematical thinking to emerge coherently across multiple forms of evidence.

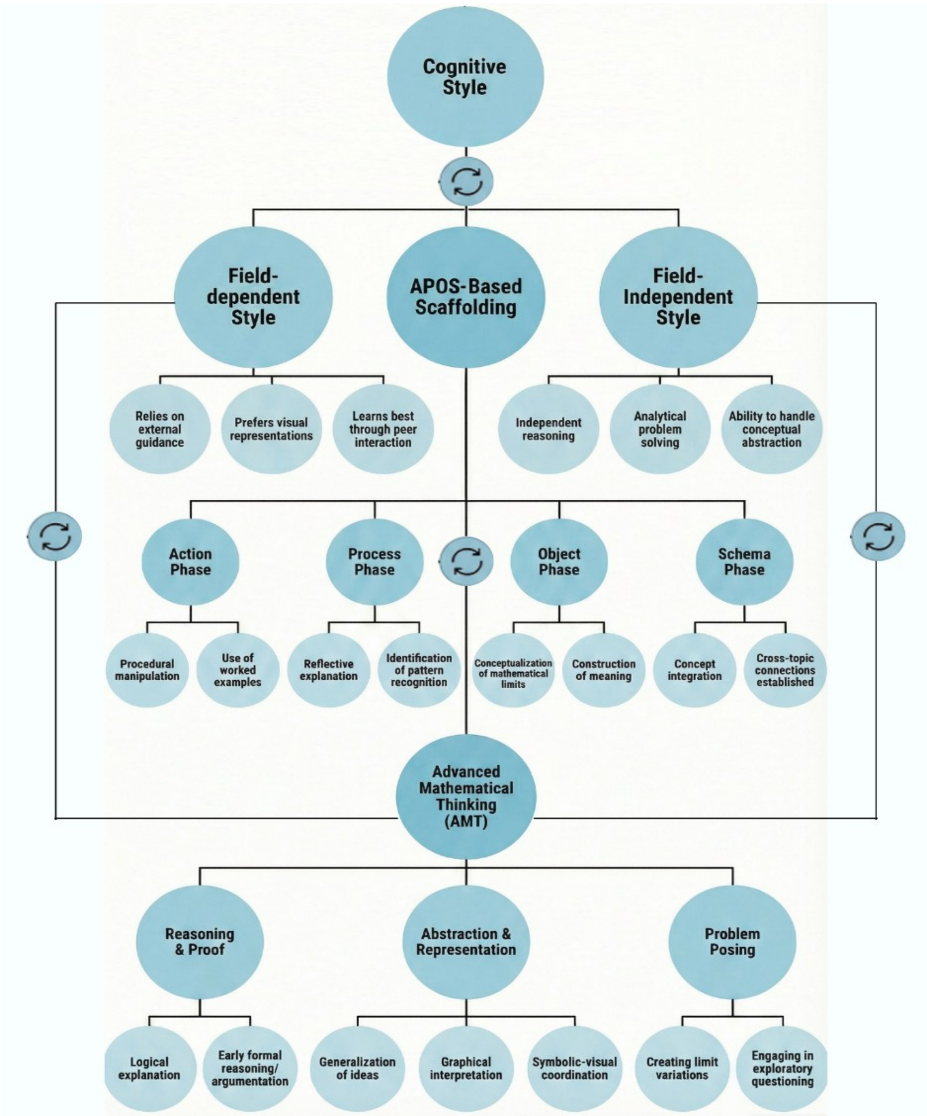

Figure 3. NVivo-Based Thematic Model of Cognitive Style, APOS-Based Scaffolding, and the Development of Advanced Mathematical Thinking (AMT) in Learning Function Limits

This conceptual diagram was developed to visualize the synthesized qualitative findings generated through NVivo-assisted thematic analysis, particularly to illustrate how students' cognitive styles and APOS-based scaffolding interact in shaping *Advanced Mathematical Thinking* (AMT) in learning the concept of function limits. Rather than representing a linear or causal instructional model, the diagram portrays a dynamic and interpretive process of meaning-making. Each component reflects categories and subcategories derived from systematic coding of interview transcripts, classroom observations, and learning documents, emphasizing that the development of AMT emerges from reciprocal relationships among learners' cognitive characteristics, staged APOS scaffolding, and individual as well as collaborative learning activities.

The NVivo visualizations indicate that *cognitive style* functions as an initial interpretive lens through which students perceive and engage with mathematical tasks during APOS-based learning. Students with a *field-dependent* cognitive style are strongly associated with nodes related to reliance on external guidance, preference for visual representations, and learning through peer interaction, particularly during the Action and *Process* phases, where procedural manipulation and pattern recognition are prominent. In contrast, *field-independent* students are more closely linked to nodes emphasizing independent reasoning, analytical problem solving, and conceptual abstraction, especially at the *Object* and *Schema* phases. The coding hierarchy further reveals strong connections between APOS stages and AMT indicators: early APOS phases support representational understanding and reflective explanation, while later phases facilitate abstraction, concept integration, and cross-topic connections. Nodes representing AMT dimensions—such as *reasoning and proof*, *abstraction and representation*, and *problem posing*—emerge as outcomes of sustained interaction between cognitive style and scaffolding practices rather than as immediate products of isolated instructional techniques. Overall, the NVivo results highlight that AMT develops as a non-linear, iterative process of conceptual construction, in which cognitive diversity shapes distinct yet equally meaningful pathways toward advanced mathematical understanding when supported by adaptive APOS-based scaffolding.

Discussion

The findings of this study suggest that the development of Advanced Mathematical Thinking (AMT) in learning function limits cannot be understood solely as the outcome of instructional exposure, but rather as a dynamic interaction between students' cognitive tendencies, classroom scaffolding, and opportunities for conceptual negotiation. Rather than merely showing that students with different cognitive styles performed differently, the study provides interpretive evidence regarding how these differences shaped students' engagement with each stage of APOS-oriented learning. In this sense, the findings support and extend prior research on APOS theory by demonstrating that the effectiveness of conceptual scaffolding is mediated by the ways students process mathematical information and organize conceptual relationships during learning activities.

The stronger performance of field-independent (FI) students in abstraction and proof-related activities appears theoretically connected to their tendency toward analytical restructuring and internal cognitive regulation. During the Object and Schema stages, FI students were more likely to reinterpret procedural actions as conceptual entities, construct symbolic relationships independently, and justify mathematical reasoning without extensive external prompts. These findings support previous international studies indicating that field-independent learners generally demonstrate stronger abstraction, reflective reasoning, and conceptual flexibility in mathematics learning (Huincahue et al., 2021; Haciomeroglu, 2015). However, the present study extends this literature by locating these cognitive differences specifically within APOS stages. The findings suggest that FI students may progress more effectively toward schema construction because they are better able to internalize mathematical operations and reorganize them into interconnected conceptual structures. This interpretation is consistent with constructivist perspectives arguing that abstraction develops through reflective reorganization of mental structures rather than through procedural repetition alone (Tall, 2013).

At the same time, the findings challenge deterministic interpretations of cognitive style that position field-dependent (FD) students as inherently weaker mathematical thinkers. The evidence instead indicates that FD students demonstrated meaningful conceptual development when scaffolding incorporated visual mediation, collaborative explanation, and guided questioning. Their stronger engagement during the Action and Process stages suggests that externally structured interaction helped reduce cognitive overload and supported gradual conceptual internalization. This finding both supports and refines earlier research showing that field-dependent learners benefit more from socially mediated and contextually supported learning environments (Pitta-Pantazi & Christou, 2009; Mefoh et al., 2021). The present study adds nuance by illustrating how collaborative scaffolding functioned not merely as social interaction, but as a mechanism for translating abstract limit concepts into accessible representations that FD students could progressively reinterpret. Thus, the findings extend previous cognitive-style research by demonstrating that the relationship between cognitive style and mathematical thinking is context-sensitive and pedagogically mediated rather than fixed.

The discussion of APOS-based scaffolding in this study also contributes theoretically to mathematics education research by illustrating that APOS stages may operate differently depending on students' cognitive processing tendencies. Previous APOS studies have generally emphasized conceptual progression from action to schema as a relatively universal developmental sequence (Leng et al., 2023; Gueudet & Winsløw, 2022). While the present findings support this theoretical progression, they further indicate that students do not experience each APOS stage uniformly. FI students appeared to benefit more from reflective prompts and proof-oriented tasks during the Object and Schema stages because these stages demanded autonomous abstraction and symbolic restructuring. In contrast, FD students relied more heavily on guided examples and collaborative explanation during earlier stages before they could engage in conceptual generalization. This finding expands earlier APOS research by suggesting that conceptual transitions within APOS are shaped not only by instructional sequencing but also by differences in cognitive orientation and learning mediation.

Another important interpretation emerging from the findings concerns the uneven development of AMT indicators. Reasoning and representation developed more prominently than proof and problem posing, indicating that students initially relied on explanatory and representational thinking before constructing formal mathematical justification. This finding supports prior international studies reporting that abstraction and informal reasoning often emerge earlier than formal proof construction in undergraduate mathematics learning (Suriyah et al., 2022; Rahayu & Agoestanto, 2023). However, the present study further suggests that this developmental pattern may reflect the epistemological complexity of proof itself. Constructing proof requires not only conceptual understanding but also confidence in formal argumentation, symbolic precision, and familiarity with deductive reasoning practices. Consequently, students who were able to interpret limits conceptually did not necessarily demonstrate immediate proficiency in formal justification. This interpretation aligns with recent scholarship emphasizing that proof development in higher mathematics is gradual, recursive, and socially mediated rather than linear or automatic (Stylianides & Stylianides, 2022).

Although cognitive style emerged as an important interpretive lens, the findings also indicate that differences in AMT development cannot be attributed exclusively to cognitive style alone. Several alternative explanations may also have contributed to the observed patterns. For example, students' prior mathematical achievement likely influenced their readiness to engage with abstraction and proof-oriented tasks. Similarly, self-confidence and mathematical self-efficacy may have shaped students' willingness to participate in collaborative discussion or attempt conceptual explanations during classroom interaction. Students with previous experience in collaborative learning environments may also have adapted more easily to APOS-oriented discussion activities than those accustomed to teacher-centered instruction. These possibilities suggest that cognitive style should be interpreted as one contributing factor within a broader network of cognitive, affective, and sociocultural influences on mathematical learning.

From a broader pedagogical perspective, the findings contribute to contemporary discussions on adaptive and inclusive mathematics instruction in higher education. The study supports recent international scholarship arguing that effective mathematics teaching requires sensitivity to learners' cognitive diversity and conceptual trajectories rather than uniform instructional delivery (Bakker et al., 2015). In particular, the findings indicate that APOS-based scaffolding becomes more pedagogically meaningful when instructional support is aligned with students' cognitive processing tendencies. For higher mathematics learning contexts, especially in Islamic higher education institutions where students often demonstrate diverse academic preparation, such adaptive scaffolding may help bridge the gap between procedural performance and conceptual understanding. Theoretically, the study contributes by positioning cognitive style not merely as an individual learner characteristic, but as an interpretive lens for understanding how students engage differently with stages of conceptual mathematical development.

Nevertheless, several limitations should be acknowledged when interpreting these findings. First, the study was conducted within a single institutional context involving a relatively small number of participants; therefore, the findings are context-specific and should not be generalized broadly to all higher mathematics classrooms. Second, although the GEFT instrument was used to classify cognitive style, such categorization may not fully capture the complexity and fluidity of students' cognitive processing tendencies. Third, the qualitative interpretation of classroom interaction and AMT indicators inevitably involves researcher subjectivity despite efforts to strengthen trustworthiness through triangulation, peer debriefing, and member checking. Finally, the study focused specifically on function limits within APOS-oriented instruction; therefore, the findings may differ in other mathematical domains or instructional environments. Future studies may therefore expand this work by examining longitudinal development of AMT across multiple mathematical topics, integrating mixed-methods approaches, or exploring additional learner variables such as metacognition, self-efficacy, and prior conceptual knowledge.

CONCLUSION

This qualitative case study indicates that the development of Advanced Mathematical Thinking (AMT) in learning function limits was manifested through the interaction between students' cognitive styles and APOS-based scaffolding within the context of a Differential Calculus classroom at Universitas Islam An Nur Lampung. The findings suggest that students with field-independent (FI) and field-dependent (FD) cognitive tendencies responded differently to APOS-oriented learning activities during the development of reasoning, abstraction, representation, proof, and problem-posing abilities. FI students appeared to engage more independently in abstraction and logical justification, particularly during the Object and Schema stages, whereas FD students benefited more from visual mediation, guided explanation, and collaborative discussion during the Action and Process stages. These differences should not be interpreted as fixed distinctions in mathematical ability, but rather as variations in how students engaged with conceptual meaning-making processes. A central contribution of this study lies in demonstrating that APOS-based scaffolding was experienced differently by students with different cognitive styles, resulting in distinct pathways in the development of AMT. The findings therefore extend existing APOS-based research by positioning cognitive style as an interpretive lens for understanding how students engage with conceptual mathematical development during classroom learning. In particular, the study suggests that the progression from Action to Schema did not occur uniformly across learners, but was mediated by differences in cognitive processing tendencies and forms of instructional support. The study also indicates that reasoning, abstraction, and representation developed more visibly than proof-related thinking, suggesting that students tended to construct conceptual understanding gradually before engaging in formal mathematical justification.

These findings support the view that AMT development in higher mathematics learning is iterative, reflective, and context-dependent rather than immediate or linear. From a practical perspective, the findings may provide useful insights for mathematics lecturers working in similar higher education contexts, particularly in calculus classrooms characterized by diverse cognitive profiles among students. The study suggests that scaffolding practices integrating reflective questioning, visual representation, collaborative discussion, and conceptual prompting may support more meaningful engagement with abstract mathematical concepts. However, because this study was conducted within a single institutional setting using a qualitative case study approach, the findings should be interpreted contextually rather than generalized broadly to all mathematics education settings.

RECOMMENDATION

Based on the scope and qualitative nature of this study, several recommendations are proposed for future research and practice. Future studies may consider involving larger and more diverse participant groups across multiple institutions to explore how the interaction between cognitive style and APOS-based scaffolding unfolds in different educational contexts. Employing a longitudinal qualitative design would also allow researchers to examine the sustained development of AMT over time, particularly in relation to proof construction and conceptual integration, which appeared to develop more slowly in this study. Further research could explore the implementation of APOS-based scaffolding in online or hybrid learning environments, where digital representations and interaction patterns may differently mediate students' cognitive engagement. Incorporating complementary qualitative techniques, such as think-aloud protocols or learning journals, may provide deeper insight into students' real-time reasoning processes and how cognitive styles dynamically influence meaning-making. Through these directions, future research can contribute to a more comprehensive understanding of how cognitive and pedagogical factors interact to support the development of Advanced Mathematical Thinking in mathematics education.

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AUTHOR CONTRIBUTIONS STATEMENT

Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
Ayu Aristika	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓		✓	
Wardono		✓				✓		✓	✓	✓	✓	✓		
Scolastika Mariani	✓		✓				✓	✓					✓	
Walid		✓			✓	✓		✓			✓	✓		

CONFLICT OF INTEREST STATEMENT

The authors declare that there is no conflict of interest.

DATA AVAILABILITY

The data that support the findings of this study can be obtained from the corresponding author upon reasonable request.

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