



## Representation Obstacles in Semantic Processes for Geometric Problem Solving

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### Abstract

Mathematics is a fundamental subject in the curriculum, but many students still struggle when it comes to representing, interpreting, and connecting mathematical information in solving geometry problems. Previous research has generally addressed learning obstacles from conceptual, procedural, didactic, or epistemological perspectives, but has not specifically examined how representational obstacles emerge within the semantic process as students construct mathematical meaning. This study addresses this gap. The aim of this research is to describe the representational obstacles experienced by students during the semantic process of solving geometric problems. In this study, the semantic process is defined as a series of activities to construct meaning through reading, sorting information, identifying keywords, connecting concepts, constructing arguments, verifying solution steps, and drawing conclusions in the form of visual, symbolic, and verbal representations. The study employed an exploratory qualitative approach involving 51 students grouped into high, medium, and low ability levels. The basis for identifying representational obstacles was established through an analysis of geometric problem-solving test results, written work traces, and interviews across five semantic stages: sequencing, identification, argument formulation, verification, and conclusion, taking into account inaccuracies, incompleteness, or failures in inter-representational transformation. The research results indicate that high-ability subjects tend to experience visual, symbolic, and verbal obstacles in the early stages, whereas subjects with moderate and low abilities predominantly experience verbal and symbolic obstacles. In the verification stage, all groups exhibited relatively similar symbolic and verbal obstacles, indicating a common difficulty in formalizing and testing the validity of solutions. Theoretically, this study reinforces the understanding that representational obstacles are not only related to conceptual mastery but also to failures in the meaning-making process during the semantic stage. Practically, these findings provide a foundation for developing more targeted geometry learning strategies to strengthen inter-representational translation, mathematical argumentation, and solution validation.

**Keywords:** Representational obstacles; Semantic process; Geometric Problem solving; Inter-representational transformation; Mathematical representation.

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## INTRODUCTION

Mathematics learning is essentially the process of developing a way of thinking that can be used to solve problems, where problem-solving is at the core of mathematics learning. Sa'dijah et al. (2020) state that problem-solving is a core activity in mathematics learning. The goal of mathematics instruction is to develop mathematical skills, which include reasoning, problem-solving, communication, making connections, and representation (NCTM, 2000). One of the important mathematical skills in mathematics learning is representation. Representation is the process of presenting notations, symbols, tables, images, graphs, diagrams, equations, or other mathematical expressions in another form (Lestari & Yudhanegara, 2015).

Representations can be viewed from two perspectives: internal and external. The internal view of representation sees representation as a cognitive process within a person or individual. Meanwhile, the external view of representation is defined as a view of representation that can be physically observed (Sternberg, 2021). External representation certainly cannot be separated from internal representation, because what a person presents in their external representation is a manifestation of their internal representation in their mind. Representations in the form of numbers, algebraic equations, graphs, tables, and diagrams are external forms of mathematical concepts. Mathematical representation skills cannot be separated from the semantic process involved in every problem to be solved.

Students' mathematical representation skills are demonstrated through their semantic processes when faced with problems to be solved (Murphy, 2015). Semantic processes involve interpreting a form of representation in the form of words, symbols, or graphics. In addition to interpreting, translating a form of representation is also a semantic process aimed at preserving meaning even across different representations (Alcock & Inglis, 2009). In the semantic process, students are trained to communicate their ideas orally and in writing so that these can serve as a basis for drawing conclusions. The semantic process cannot stand alone because, in principle, it involves the semantic structures possessed by the problem-solver. In the semantic process, students sometimes encounter obstacles in understanding problems. The problems students face when solving problems are called learning obstacles.

Students' difficulties in answering math problems are often not followed up by tracking the sources of these difficulties during the learning process. Furthermore, it appears that instructors' perspectives on the sources of students' learning difficulties have received insufficient attention in research and the mathematics education literature. In this regard, instructors must help students develop perseverance and a broader perspective on mathematics (Schackow & Thompson, 2005). In the college curriculum, geometry is a core subject that hones mastery of concepts and theories acquired systematically through reasoning during the learning process. Geometry deals with abstract concepts represented by symbols. In geometry, there are also definitions, axioms, and theorems to demonstrate the truth of a statement to be proven, as stated by Chazan & Lueke (2009). A preliminary study was conducted to obtain an initial overview of this research idea. In September 2025, the researcher posed questions related to plane figures to students in the Mathematics Education Program at Insan Budi Utomo University. One of the responses from three students exhibited semantic representation difficulties, as shown in Figure 1.

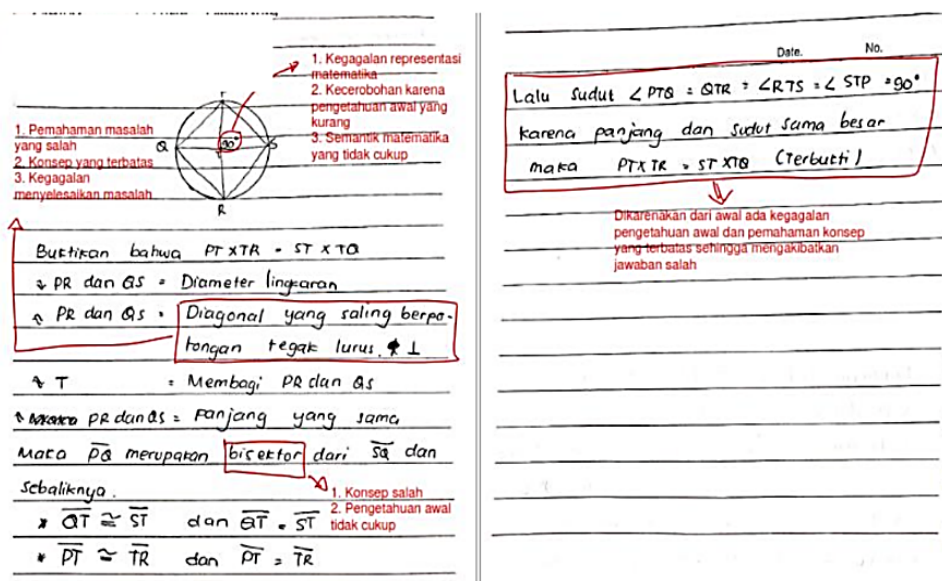


Figure 1. Responses from participants who encountered difficulties

The urgency of this issue is evident from a preliminary study conducted among students in the Mathematics Education Program at Insan Budi Utomo University. In the geometry assignment given, it was found that students' answers appeared complete at first glance because they included drawings, symbols, and written explanations; however, they actually revealed difficulties with representation in the semantic process. Students created inaccurate visualizations, wrote symbols that did not fully correspond to the intended geometric relationships, and misinterpreted the questions, resulting in chosen solution strategies that did not lead to correct proofs or results. These initial findings indicate that the presence of representations does not guarantee that students have constructed correct mathematical meaning. The issue is not merely "being unable to draw" or "calculating incorrectly," but rather the existence of obstacles when students interpret, connect, and validate representations within the semantic flow of problem-solving. This results in failure to solve the problem. Table 1. shows a comparison of the research positions based on the research that has been conducted.

**Table 1.** Position of this study compared to previous studies

<b>Research Focus</b>	<b>Previous Studies</b>	<b>Main Findings of Previous Studies</b>	<b>Contribution to the Present Study</b>	<b>Position of the Present Study</b>
Learning obstacles	Fauzi (2020); Kurniawan et al. (2017)	Previous studies identified three types of learning obstacles, namely ontogenetic, epistemological, and didactic obstacles. These obstacles are influenced by internal and external factors.	These studies provide a theoretical basis for understanding the types and sources of students' learning obstacles.	The present study does not merely identify general learning obstacles, but focuses specifically on obstacles in students' mathematical semantic representation when solving geometry problems.
Semantics in mathematical problem solving	Chiu et al. (2014)	Semantic research focused on exploring mathematical relationships from explicit problems in arithmetic and geometry. Semantics was described through four components: parsing and annotation, entity identification, relation extraction, and equation instantiation.	This study contributes to semantic analysis through: a problem-understanding approach based on equivalent relationships, a semantic syntax model, and explicit extraction of relationships from arithmetic and geometry word problems.	The present study adopts the semantic perspective, but applies it to identify students' obstacles in representing mathematical meaning, especially in geometry problem solving.
Mathematical representation	Ernaningsih & Wicasari (2017)	The study reported three types of representation: icon representation, verbal representation, and visual representation.	This study provides a basis for understanding how students express mathematical ideas through different forms of representation.	The present study narrows the representation focus to mathematical semantic representation, which concerns how students

<b>Research Focus</b>	<b>Previous Studies</b>	<b>Main Findings of Previous Studies</b>	<b>Contribution to the Present Study</b>	<b>Position of the Present Study</b>
				understand, interpret, and express mathematical meaning in geometry problems.
Cognitive obstacles in representation	Murniasih et al. (2020)	The study identified several cognitive obstacles, including verbal representation, the tendency to generalize, reliance on intuition, strategies, and less meaningful learning. These aspects were defined as obstacles in students' mathematical thinking.	This study shows that obstacles may emerge not only from content difficulty, but also from students' cognitive tendencies and representational limitations.	The present study extends this perspective by examining mathematical semantic representation as an obstacle that affects students' ability to solve geometry problems.
Preliminary study and present research	Preliminary study by the researchers	The preliminary study showed that mathematical semantic representation becomes an obstacle in solving geometry problems.	The preliminary findings strengthen the need to investigate semantic representation as a specific form of obstacle in mathematics learning.	Based on previous studies and preliminary findings, this study aims to identify obstacles to students' mathematical semantic representation in solving mathematical geometry problems.

Based on Table 1. several studies have discussed obstacles (Fauzi, 2020; Kurniawan et al., 2017). In these studies, we observed three types of obstacles, namely ontogenetic, epistemological, and didactic, which are influenced by internal and external factors. Semantic research (Chiu et al., 2014) focuses on exploring mathematical relationships from explicit problems in arithmetic and geometry. In addition, in this study, semantics is described through four components: parsing and annotation, entity identification, relation extraction, and equation instantiation. Previous research results provide three important contributions to semantics, namely (1) a new approach to understanding problems that extracts equivalent relationships representing problems in finding solutions, (2) a semantic syntax model, and (3) a proposal to explicitly extract relationships from explicit word problems in arithmetic and geometry. Research on representation (Ernaningsih & Wicasari, 2017) reports three types of representation, namely icon representation, verbal representation, and visual representation. Research on cognitive obstacles (Murniasih et al., 2020) identified verbal representation, the tendency to generalize, the tendency to rely on intuition, strategies, and less meaningful learning, all of which are defined as obstacles. Our preliminary study shows that mathematical semantic representation is a obstacles in solving geometry problems. Based on previous studies

and preliminary research, this study was conducted to identify obstacles to mathematical semantic representation in solving mathematical geometry problems.

Previous studies have shown that learning obstacles can vary in type. identified three types of obstacles, namely conceptual, procedural, and operational. Fauzi (2020) identified three types of obstacles, namely ontogenetic, epistemological, and didactic obstacles. There are differences in the types of obstacles identified in these two studies. Jannah (2019) identified four types of obstacles in her study, namely cognitive, genetic & psychological, didactic, and epistemological. Although there are differences in the types of obstacles identified in these three studies, none of them examined mathematical semantic representation. Previous studies only discussed obstacles, but this study will look at obstacles in the process of semantic representation. The researcher also noted the lack of research on mathematical semantics, which is the reason for choosing the theme of mathematical semantics. The combination of obstacles, representation, and semantics adds to the novelty of this study.

Based on this gap, the novelty of this study lies not only in the integration of the three concepts obstacles, representation, and semantics but also in the effort to explicitly map representational obstacles at each stage of the semantic process in solving geometric problems, namely sequencing, identification, argument formulation, verification, and conclusion. This study also treats visual, symbolic, and verbal representations not as separate categories, but as interconnected parts of the process of mathematical meaning making. In this way, the study aims to demonstrate more clearly how obstacles emerge, shift, or recur from one stage to the next, as well as how these patterns of obstacles differ according to the subjects' ability levels. This novelty is significant because it offers a deeper analytical perspective compared to previous studies, which tended to separate the analysis of obstacles, the analysis of representations, and the analysis of semantics. On this basis, this study aims to describe the representational obstacles experienced by students in the semantic process when solving geometry problems. This focus is expected to provide a theoretical contribution in the form of a more integrated framework for understanding students' difficulties in geometry, as well as a practical contribution to the design of instruction that is more attuned to points of conceptual breakdown in mathematical problem-solving.

## **METHOD**

### **Research Design**

This study employed an exploratory qualitative approach to describe representational obstacles in students' semantic processes when solving geometry problems. This approach was used because the study focused on examining students' reasoning, written work, and verbal explanations in depth rather than measuring the effect of a particular treatment. The researcher acted as the key instrument in collecting, interpreting, and analyzing the data. The analysis was directed toward identifying how students constructed mathematical meaning through visual, symbolic, and verbal representations across five semantic stages, namely sorting, identification, argument formulation, verification, and conclusion (Creswell, 2012).

### **Research Setting and Participants**

This study was conducted in the Mathematics Education Program, Faculty of Exact Sciences and Physical Education, Insan Budi Utomo University, with students from the 2025 cohort taking the Euclidean Geometry course. Research subjects were selected based on the results of a problem-solving test that had been designed, developed, and validated. The problem-solving test instrument was designed based on the material from the Euclidean Geometry course. The selected subjects were those who exhibited both existing and new types of difficulties.

The selection criteria were: (1) Students were selected based on their abilities; (2) students were selected based on their semantic representations and observed difficulties, as well as their ability to communicate their opinions and ideas verbally and in writing. To

determine this, the researcher sought input from the instructors teaching those classes. In this study, the researcher measured students' mathematical ability using the Geometry Problem-Solving Test so that students could be classified based on their level of mathematical ability, namely high ability, moderate ability, and low ability. The score intervals for each ability level can be seen in Table 2 below.

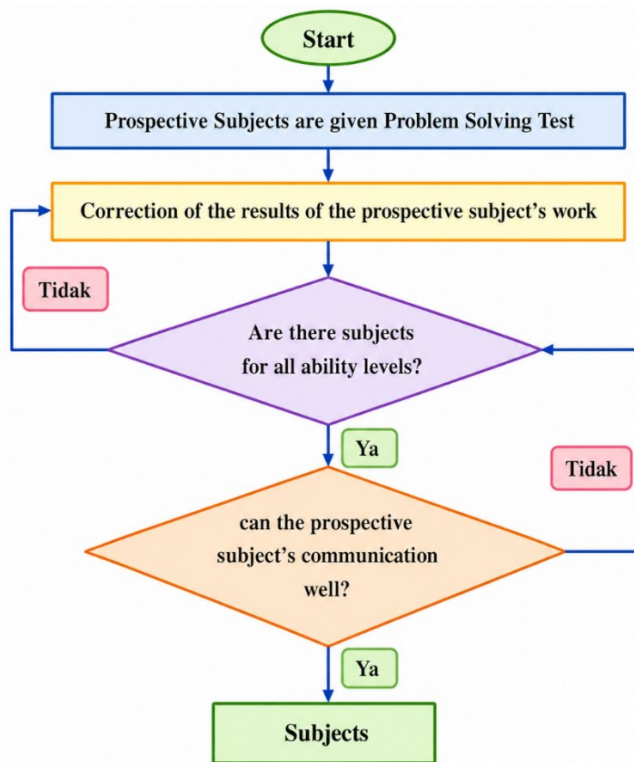
**Table 2.** Value Intervals for Each Ability Level

Value(x)	level of ability
$0 \leq x < 60$	Low
$60 \leq x < 80$	Medium
$x \geq 80$	High

The initial study participants consisted of 51 students, all of whom took the Geometry Problem-Solving Test (GPST). The test results were used to categorize the students' mathematical abilities into three categories: high, moderate, and low. Ability categories were determined based on test scores using the following criteria: high for scores  $\geq 80$ , moderate for scores 60–79, and low for scores  $< 60$ . This grouping was used practically to obtain a variety of subject characteristics and was theoretically based on the needs of qualitative research to compare patterns of representational obstacles at different ability levels. Of the 51 students, 14 were categorized as high, 24 as moderate, and 13 as low.

**Subject Selection and Code Description**

The subject selection procedure was conducted in two stages. First, all 51 students were analyzed during the initial mapping phase based on their written responses to the Geometry Problem-Solving Test. The test results were used to classify students into three mathematical ability categories, namely high, moderate, and low. Second, an in-depth qualitative analysis was conducted on three selected subjects, with one subject representing each ability category. Subject selection was carried out using purposive sampling, as illustrated in Figure 2.



**Figure 2.** Subject selection flowchart

The subject selection flowchart shows the process of selecting subjects who experienced semantic representation obstacles in solving geometry problems. The selection criteria

included: (1) representing a specific mathematical ability category based on the Geometry Problem-Solving Test scores; (2) showing indications of visual, symbolic, or verbal representation obstacles in written responses; and (3) being able to articulate thought processes verbally during the interview. The selected subjects were determined through a review of the test results, followed by consultation with the course instructor to ensure that they were communicative and willing to participate in in-depth interviews. Thus, all 51 participants were used for the initial classification, whereas the focus of the in-depth qualitative analysis was directed toward three purposively selected subjects.

To support clarity in the Results and Discussion sections, the selected subjects were assigned specific codes. The high-ability subject was coded as ST, the moderate-ability subject was coded as SS, and the low-ability subject was coded as SR. These codes are used consistently in the presentation of written work, interview excerpts, and analysis of representational obstacles. In the interview transcripts, the code P refers to the interviewer or researcher. Meanwhile, additional codes attached to students' statements, such as STPMKG01, SSIMKG01, or SRVPM01, function as data-tracking codes. These codes were used to organize excerpts according to the subject, semantic stage, representational focus, and sequence of data. Therefore, these excerpt codes do not indicate different respondents, but serve to ensure systematic data organization and traceability during analysis.

The characteristics of the subjects were also considered to clarify the context of the study. All participants were students who were taking the Euclidean Geometry course in the current semester. They had studied relevant prerequisite material, including plane figures, similarity, area, and relationships between geometric elements. Because the research was conducted in the context of geometry learning in higher education, the identified representation obstacles are understood within the framework of formal problem solving among prospective mathematics teachers.

**Research Instruments**

The primary research instruments were the Problem-Solving Test (PST), which had been validated by experts, and a structured interview guide designed to explore more in-depth information about the semantic processes at each stage of problem-solving. Prior to use, the validity of the Geometry Problem-Solving Test instrument was assessed through expert judgment by two validators: a faculty member holding a doctoral degree in mathematics education and a faculty member specializing in mathematics/geometry. Validation covered four aspects: the alignment of item content with research objectives, clarity of language, the appropriateness of item construction to elicit semantic representation processes, and the alignment of items with students' ability levels.

The validation results indicated that the instrument is suitable for use with revisions. The revisions included simplifying the wording, changing the question context to make it easier to understand without altering the cognitive demands, and adjusting the phrasing of the questions to more effectively elicit representational responses. For example, the original geometry proof question was revised into a contextual problem regarding the area of the largest square within a right triangle, as this revision was deemed more effective for gradually eliciting students' visual, symbolic, and verbal representations. Through this process, the instrument was deemed adequate for capturing the construct of representational obstacles in the semantic process. Based on the validation results, it was concluded that the Geometry Problem-Solving Test instrument designed by the researcher is suitable for use with the revisions. However, these revisions do not alter the underlying content but only modify the questions to be tested.

**Table 3.** Expert Revision Validation Results

Before	After
In a circle, there is a quadrilateral formed by chords. PQRS is the quadrilateral formed by	A triangular yard has sides measuring 9 m, 12 m, and 15 m. Rey is going to lay a square carpet

Before	After
chords, and T is the intersection point of its diagonals. Prove that $PT \times TR = ST \times TQ!$	in the yard. What is the largest square area that Rey can cover with the carpet?

### Data Collection Procedure

Data collection was conducted in two phases. The first phase involved administering the GPST to all 51 students. The students' responses were corrected, scored, and then grouped according to ability categories. In addition to the scores, the researchers also analyzed the written responses to identify early indications of representational difficulties. The second stage involved in-depth interviews with three selected subjects. The interviews were conducted individually, using a semi-structured guide, and focused on exploring the reasoning behind each step of the solution, the forms of representation used, and the difficulties the subjects encountered at each stage of the semantic process. All interviews were recorded and then transcribed verbatim for further analysis.

### Data Analysis

Data analysis was conducted through data reduction, data presentation, and conclusion drawing. In the reduction stage, the researcher selected relevant data from students' written responses and interview transcripts. The data were then organized according to the five semantic stages: sorting, identification, argument formulation, verification, and conclusion. At each stage, the researcher identified the forms of representation used by the subjects and the obstacles that appeared in their reasoning.

Visual representation obstacles were identified when subjects were unable or inaccurate in presenting geometric situations through drawings, sketches, or spatial relationships. Symbolic representation obstacles were identified when subjects made errors in using notation, variables, formulas, equations, or mathematical operations. Verbal representation obstacles were identified when subjects were unable to clearly explain the meaning of the problem, justify their strategy, connect mathematical concepts, or formulate an appropriate conclusion.

The analysis was conducted by comparing the subjects' written work with their interview responses. The findings were then interpreted to describe how representational obstacles emerged at each semantic stage and how these obstacles differed among ST, SS, and SR. This procedure enabled the researcher to identify not only the types of obstacles experienced by each subject, but also the points in the semantic process where the obstacles occurred.

### Criteria for Identifying Representation Obstacles

The criteria for identifying representation difficulties were explicitly defined to ensure a more systematic analysis. Visual representation difficulties are defined as a subject's inability or inaccuracy in presenting geometric situations through drawings, sketches, or relevant spatial relationships. Symbolic representation obstacles are defined as the inability or error in using notation, variables, formulas, equations, or mathematical operations to model and solve problems. Verbal representation obstacles are defined as the subject's inability or lack of clarity in explaining the intent of a problem, the rationale for strategy selection, relationships between concepts, or final conclusions, both orally and in writing. These three criteria are applied consistently at every stage of the semantic process so that it can be determined at which stage a particular obstacles arises and how the differences between ability categories manifest.

To ensure the validity of the data, this study employed methodological triangulation, which involved comparing data from written tests with data from interviews. Additionally, the researcher also checked the consistency of interpretations by re-examining the alignment between written responses, the subjects' verbal statements, and the established indicators of obstacles. Through this procedure, the analysis results are expected to depict the obstacles to representation more accurately and be methodologically accountable.

**RESULTS AND DISCUSSION**

This study aims to qualitatively describe the semantic representation obstacles students face when solving geometry problems. A total of 51 students participated in a problem-solving test and were subsequently classified into three ability groups: high (14 students), moderate (24 students), and low (13 students). This study identified obstacles to representation in students' semantic processes when solving geometry problems. The subjects went through semantic stages ranging from sequencing, identification, formulation of arguments, verification, and conclusion.

**High-Ability Students (ST)**

In general, high ability subjects have obstacles in almost all types of representation at various semantic stages. Although they are able to start the solution well, obstacles arise during the technical and strategic processes.

- a. Sorting Stage: Subjects are able to understand information and visualize problems in the form of images. However, obstacles arise in the form of errors in determining the length of the sides of the triangle shown in the image.
- b. Identification Stage: Subjects successfully identify the keyword "triangle with a square inside," but experience obstacles in interpreting the keyword because they fail to determine the length of the sides of the square.
- c. Argument Formulation Stage: The dominant obstacle is the inability to find the length of the sides of the square using the concept of similarity. Although the subject understands the basic concept of similarity, the initial error in determining the sides causes the comparison of sides to be incorrect.
- d. Verification Stage: The subject used algebraic concepts (quadratic equations) to find the value of the sides, but made a calculation error in the algebraic division operation, resulting in an inaccurate final result.
- e. Conclusion Stage: The subject was able to draw a final conclusion about the area of the square, but was unsure of the answer because he realized that there were inaccuracies in the strategy he used.

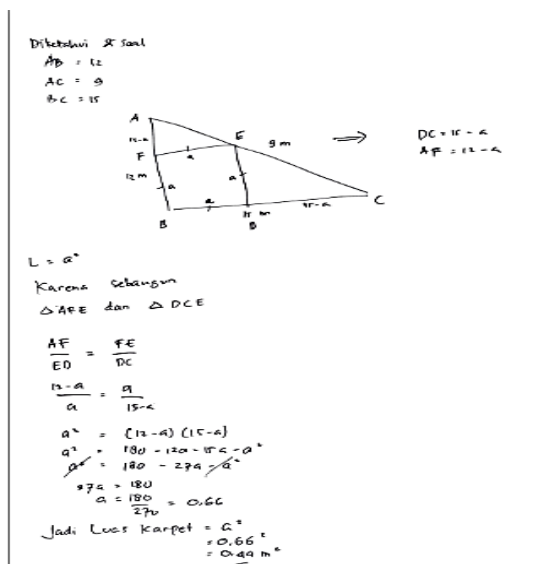


Figure 3. Answer Sheet for Subject ST

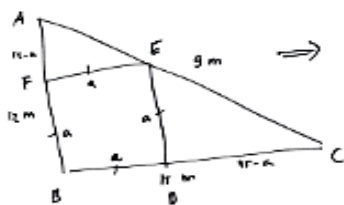
**Interview Transcript**

**Sorting Stage**

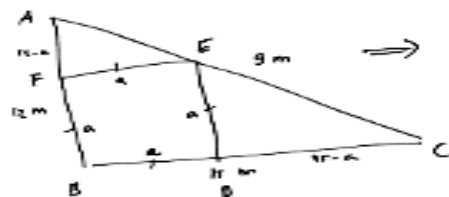
- P : After reading the problem, what do you understand from the problem presented? Symbolic, Verbal
- ST : From the problem, the sides of the triangle are known, Ma'am. STPMKG01



- P : Which information in the problem led you to that understanding? Symbolic, Verbal
- ST : This part, Ma'am: a house yard is triangular in shape, with side lengths of 9 m, 12 m, and 15 m. STP02
- P : Is there any information given that you do not yet understand? Symbolic, Verbal
- ST : I understand everything, Ma'am. STP03
- P : What do you think about, or what will you do next, after reading the problem? Visual, Symbolic
- ST : I think it would be easier if I draw it, Ma'am. STPMKG04



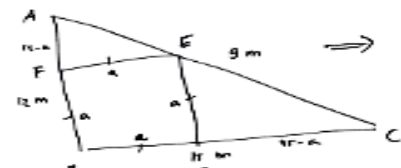
- P : Is there any information you need to add to express your understanding and support the solution process? Visual, Symbolic
- ST : Umm... I will draw it and then write down the side lengths, Ma'am. STPMKG05



- P : Does that information support the proof process that will be carried out? Symbolic, Verbal
- ST : Yes, it supports the process because knowing the side lengths can make it easier to solve the problem. STP06

**Identification Stage**

- P : Are there any keywords that you identified from the problem? Symbolic, Verbal
- ST : Yes, a triangle with a square inside it. STIMKG01

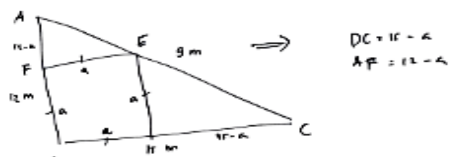


(ST points to the drawing.)

- P : Why did you choose those keywords? Symbolic, Verbal
- ST : Because they are clearly stated in the problem. STI02
- P : What is the meaning of the keywords you identified? Symbolic, Verbal
- ST : They refer to what needs to be solved. STIMKG03
- P : Can the keywords you identified support the proof process that will be carried out? Symbolic, Verbal
- ST : Yes, definitely, Ma'am. STI04

**Argument Formulation Stage**

- P : What will you do to prove the statement? Symbolic, Visual, Verbal
- ST : I will write the side lengths on the drawing that I made. STFPPA01



- P : Why do you do that? Verbal
- ST : So that it will be easier for me to answer this problem. STFA02
- P : Can you explain the steps of the proof that will be carried out? Symbolic, Verbal
- SE : After I write the side lengths on the drawing, I will then construct the proof. STFAPP03

$L = a^2$   
 Karena sebangun  
 $\triangle AFE$  dan  $\triangle DCE$

- P : Which information do you use to do that? Symbolic, Verbal
- ST : The proof that the two triangles are similar, Ma'am. STFA04

$\triangle AFE$  dan  $\triangle DCE$

- P : Why do you use that information? Symbolic, Verbal
- ST : To calculate the side lengths STFAPP05

$\triangle AFE$  dan  $\triangle DCE$

$$\frac{AF}{ED} = \frac{FE}{DC}$$

If the two figures are similar, their corresponding sides and angles are considered. The corresponding angles are equal, while the corresponding sides are not necessarily equal in length but are proportional.

- P : What form of equation did you obtain? Symbolic, Verbal
- ST : Triangle AFE and triangle DCE. STFA06

$\triangle AFE$  dan  $\triangle DCE$

**Verification Stage**

- P : How did you obtain that new conclusion or statement from the given statement? Symbolic, Verbal
- ST : From this similarity proof, the side length of the square will later be obtained. STVMKG01

$$\frac{AF}{ED} = \frac{FE}{DC}$$

$$\frac{15-a}{a} = \frac{a}{15-a}$$

- P : Are there any mathematical concepts that you use to support that conclusion or statement? Please mention them. Symbolic, Verbal
- ST : Yes, a quadratic equation. STVMKG02

$$\frac{15-a}{a} = \frac{a}{15-a}$$

$$a^2 = (15-a)(15-a)$$

$$a^2 = 180 - 12a - 15a + a^2$$

- P : Explain the algebraic operation process that you carried out. Symbolic, Verbal
- ST : I will factor this quadratic equation. STV03

$$a^2 = (15-a)(15-a)$$

$$a^2 = 180 - 12a - 15a + a^2$$

$$a^2 = 180 - 27a - a^2$$

P	: Why does that algebraic operation need to be carried out?	Symbolic, Verbal
ST	: To calculate the value of $a$ that is being sought.	STVPM03

$$\begin{aligned}
 4a &= 180 - 27a - a^2 \\
 27a &= 180 \\
 a &= \frac{180}{27} = 0,66
 \end{aligned}$$

**Conclusion Stage**

P	: What conclusion did you draw to answer the problem presented?	Symbolic, Verbal
ST	: From this value of $a$ , I will determine the area of the square.	STKPM01

$$\begin{aligned}
 a &= \frac{180}{27} = 0,66 \\
 \text{Jadi Luas Karpas} &= a^2 \\
 &= 0,66^2
 \end{aligned}$$

P	: How are you confident that the answer is correct?	Symbolic, Verbal
ST	: Actually, I am not completely sure, Ma'am.	STKPM02
P	: Is your answer the answer to the question in the problem? What is the meaning of the conclusion you made?	Symbolic, Verbal
ST	: Yes, the conclusion is that the area of the carpet to be installed is 0.44 m <sup>2</sup> .	STKPM03

$$\begin{aligned}
 \text{Jadi Luas Karpas} &= a^2 \\
 &= 0,66^2 \\
 &= 0,44 \text{ m}^2
 \end{aligned}$$

**Moderate-Ability Students (SS)**

Subjects with moderate ability exhibit more significant difficulties in conceptual and mathematical modeling aspects.

- a. Sorting Stage: Subjects recognize the presence of the “square” element in the problem but do not systematically record the known information. Difficulties arise because subjects are unable to represent the problem visually and immediately resort to using the area formula without understanding the context.
- b. Identification Stage: Keywords are limited to the word “square.” The subject experiences difficulties in connecting existing information (relationships between shapes) and focuses solely on how to calculate the area of a square.
- c. Argument Formulation Stage: A critical conceptual difficulty arises, where the subject applies the area formula for a rectangle ( $L = p \times l$ ) to a square and confuses it with the concept of perimeter. The subject is also unable to create relevant mathematical models or equations.
- d. Verification Stage: The subject experiences a serious symbolic representation obstacles by assuming that the area of a square is equal to the perimeter of the triangle calculated (36 m<sup>2</sup>). This conceptual error indicates that the subject is unable to logically verify the results of their work.
- e. Conclusion Stage: In the final stage, subjects with moderate ability tend to experience symbolic and verbal representation difficulties similar to the previous stage, namely an inability to connect the calculation results with the initial problem statement.

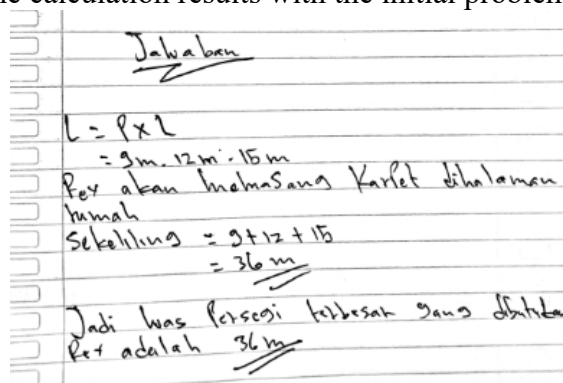
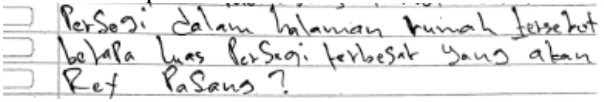
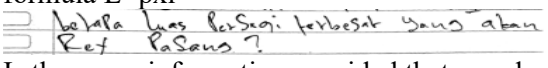
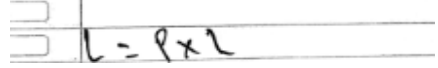
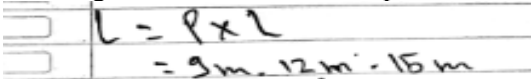


Figure 4. Answer Sheet for SS

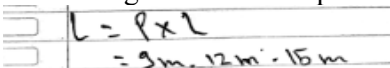
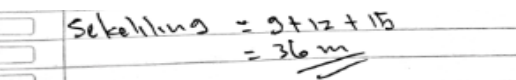
**Sorting Stage**

P	: After reading the problem, what do you understand from the problem presented?	Symbolic, Verbal
SS	: There is a square 	SSPMKG01
P	: What information led you to that conclusion?	Symbolic, Verbal
SS	: The largest square being sought is continuously substituted into the formula $L=pxl$ 	SSP02
P	: Is there any information provided that you don't understand?	Verbal
SS	: I'm still confused about what to do with the triangle	SSPMKG03
P	: Next, what are your thoughts or what will you do after reading about this issue?	Symbolic
SS	: Enter the square root formula 	SSPMKG04
P	: Is there any additional information you'd like to provide to explain your understanding and support the resolution process?	Verbal, Symbolic
SS	: The lengths of the sides of the square 	SSPMKG05
P	: Does that information support the evidentiary process that will be conducted?	Verbal
SS	: Yes	SSPMKG06

**Identification Stage**

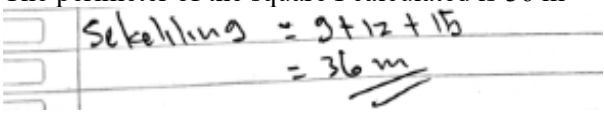
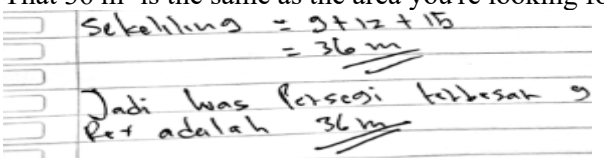
P	: Have you identified any keywords related to the issue?	Verbal
SS	: There's a square word	SSIMKG01
P	: Why did you choose those keywords?	Verbal
SS	: That is what we are looking for	SSIMKG02
P	: What is the meaning of the specified keyword?	Verbal
SS	: The rug is square shaped	SSIMKG03
P	: Will the keywords you've selected support the verification process that will be conducted?	Verbal
SS	: I'll calculate the area of that square	SSIMKG04

**Argument Formulation Stage**

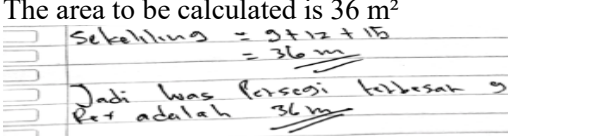
P	: What will you do to prove that statement?	Symbolic, Verbal
SS	: Calculating the area of a square using the formula length $\times$ width 	SSFAPP01
P	: Why did you do that?	Verbal
SS	: That is what we are looking for	SSFAPP02
P	: Can you explain the steps involved in the proof that will be conducted?	Symbolic, Verbal
SS	: Let's just find the area first, and then I'll calculate the perimeter. 	SSFAPP02
P	: Which instructions did you use to do that?	Verbal
SS	: Square shaped carpet	SSFA03

P	: Why did you use that explanation?	Verbal
SS	: I'm a little confused too, but that's what I can do by calculating the area of that square	SSFAPP04
P	: What does the equation you derived look like?	Symbolic, Verbal
SS	: $L = p \times l$ and perimeter	SSFAPP05

**Verification Stage**

P	: How did you arrive at that new conclusion or statement based on the given statement?	Symbolic, Verbal
SS	: The perimeter of the square I calculated is $36 \text{ m}^2$ 	SSVPP01
P	: Are there any mathematical concepts you used to support that conclusion or statement? Please list them!	Symbolic, Verbal
SS	: That $36 \text{ m}^2$ is the same as the area you're looking for, right? 	SSVPP02
P	: Please explain the algebraic operations you performed.	Verbal
SS	: I'll just equate the perimeter with the area	SSVPP03
P	: Why is it necessary to perform that algebraic operation?	Verbal
SS	: To achieve the desired results	SSVPP04

**Conclusion Stage**

P	: What conclusion did you draw to address the problem presented?	Symbolic, Verbal
SS	: The area to be calculated is $36 \text{ m}^2$ 	SSKPM01
P	: How can you be sure that the answer is correct?	Verbal
SS	: I'm not sure, ma'am	SSKPM02
P	: Is your answer the answer to the question posed in the problem? What is the significance of the conclusion you have drawn?	Verbal
SS	: I don't think so, hehehe...	SSKPM02

**Low Ability Subjects (SR)**

Based on the general summary in the document, low ability subjects have the most fundamental obstacles, especially at the conclusion stage. General Characteristics: Unlike high and medium ability subjects who face obstacles in various types of representation (symbolic and verbal), low ability subjects are mainly hindered in verbal representation at the final stage. They often fail to provide logical explanations or draw conclusions that are in line with the initial purpose of problem solving.

The findings show variations in obstacles among subjects with different ability levels. For example, high-ability subjects were found to have obstacles in all types of representation, including an inability to present problems in visual form and errors in determining side lengths or creating mathematical models (symbolic). Meanwhile, subjects with moderate abilities showed fatal obstacles such as conceptual errors in the use of area and perimeter formulas and an inability to establish relationships between the problems given. In general, failure to represent problems in the form of similarity and inappropriate strategies were the main causes of subjects' incorrect answers.

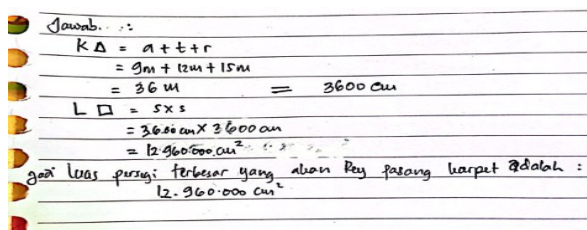


Figure 5. Answer SR

**Sorting Stage**

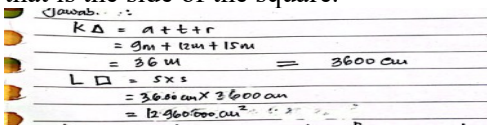
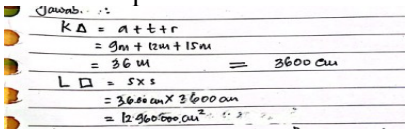
P	: After reading the problem, what do you understand from the problem presented?	Symbolic, Verbal
SR	: A triangle with sides of 9, 12, and 15	SRP01
P	: What information led you to that conclusion?	Symbolic, Verbal
SR	: In the problem, there is a triangle with three sides	SRP02
P	: Is there any information provided that you don't understand?	Symbolic, Verbal
SR	: Do you understand everything, ma'am	SRP03
P	: Next, what are your thoughts or what will you do after reading about this issue?	Symbolic
SR	: Hmm... Finding the area of a square	SRP04
P	: Is there any additional information you'd like to provide to explain your understanding and support the resolution process?	Verbal, Symbolic
SR	: I'm going to find the area of the largest square, but first I'll find the perimeter of the triangle...that's how my teacher taught me	SRPMKG05
P	: Does that information support the evidentiary process that will be conducted?	Symbolic, Verbal
SR	: Yes, I fully support it	SRPMKG06

**Identification Stage**

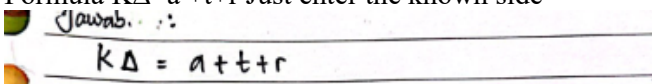
P	: Are there any keywords you've identified from the problem?	Verbal
SR	: Triangle and square	SRI01
P	: Why did you choose those keywords?	Verbal
SR	: That's all we know about the question, ma'am	SRI02
P	: What is the meaning of the specified keyword?	Verbal
SR	: What is it, ma'am? Just triangles and squares, ma'am... no particular reason	SRI03
P	: Will the keywords you've selected support the verification process that will be conducted?	Verbal
SR	: Yes	SRI04

**Argument Formulation Stage**

P	: What will you do to prove that statement?	Symbolic, Verbal
SR	: I'll calculate the perimeter first, then the area of the square	SRFAPP01
P	: Why did you do that?	Symbolic, Verbal

SR	: Since the triangle is given in the problem, I'll calculate the perimeter of the triangle first	SRFAPP02
P	: Can you explain the steps involved in the proof you're going to present?	Symbolic, Verbal
SR	: First, calculate the perimeter of the triangle; once you have the result, that is the side of the square.	SRFAPP03
		
P	: Which instructions did you use to do that?	Symbolic, Verbal
SR	: The known side is substituted into the formula	SRFA04
P	: Why did you use that explanation?	Symbolic, Verbal
SR	: To simplify the calculation	SRFA05
P	: What does the equation you derived look like?	Symbolic, Verbal
SR	: I'll just plug those values into the formulas for the perimeter and the area of a square	SRFA06
		

**Verification Stage**

P	: How did you arrive at that new conclusion or statement based on the given statement?	Symbolic, Verbal
SR	: I remember my professor once telling us that a triangle is half a square	SRVPM01
P	: Are there any mathematical concepts you used to support that conclusion or statement? Please list them!	Symbolik, Verbal
SR	: So, since the square is half the area of the triangle, I'll just calculate the perimeter first and then find the area of the square.	SRVPM02
P	: Please explain the algebraic operations you performed.	Symbolic, Verbal
SR	: Formula $K\Delta = a + t + r$ Just enter the known side	SRVPM03
		
P	: Why is it necessary to perform that algebraic operation?	Symbolic, Verbal
SR	: I already explained earlier that a triangle is half of a square	SRVPM04

**Conclusion Stage**

P	: What conclusion did you draw to address the problem presented?	Verbal
SR	: The area of a square can be found using the perimeter of a triangle	SRPPM01
P	: How can you be sure that the answer is correct?	Verbal
SR	: Yeah, to get the square with the largest area, I'll just convert from meters to centimeters and then multiply to find the largest area.	SRPPM02
P	: Is your answer the answer to the question posed in the problem? What is the significance of the conclusion you have drawn?	Verbal
SR	: I'm not really sure	SRPPM03

**Discussion**

Representational obstacles in students' semantic processes in solving geometry problems. The components of representational obstacles in the semantic processes used in this study are (1) Sorting, (2) Identification, (3) Argument Formulation, (4) Verification, and (5) Conclusion. Meanwhile, the problems used are geometry problems. A problem is said to contain incomplete

information if one or more aspects of the problem situation are not specified, the problem is not clearly described, or the information needed to solve the problem is not clearly described (Jonassen, 1997). Subjects were selected based on high, medium, and low ability levels.

The semantic process for each subject begins with four types of reading the problem to identify the problem situation presented. The initial stage of the semantic process involves sorting important information from the problem and providing notes that support the problem situation. This activity is carried out by problem solvers to identify information that supports the problem-solving process (Easdown, 2009; Hong, 1998; Panasuk & Beyranevand, 2011; Yu, et al., 2017). The goal is to find all the information needed as well as information that does not support the problem-solving process (Adu-Gyamfi, et al., 2016; Gagantis & Elia, 2005; Sajadi, et al., 2013). The process of selecting and sorting is also carried out to determine keywords, with the aim of interpreting them in accordance with the context of the problem. The process of sorting, identifying, and interpreting is in accordance with previous research (Mao & Sen, 2018; Yu et al., 2017), which is the initial process of problem solving. In the sorting process carried out by the subjects, visual representation obstacles, symbolic representation obstacles, and verbal representation obstacles emerged. Visual representation obstacles only appeared in subjects with high abilities. Meanwhile, symbolic representation obstacles and verbal representation obstacles appeared in all ability levels.

After sorting, the next step is to identify keywords. Keyword identification is the activity of identifying and interpreting keywords from the problem situation. As stated by previous researchers (Mao & Sen, 2018; Yu et al., 2017), this activity aims to identify, define, group, and interpret keywords according to the problem situation. During the identification process, subjects encountered visual representation obstacles and verbal representation obstacles. Visual representation obstacles and symbolic representation obstacles only appeared in subjects with high abilities. Meanwhile, verbal representation obstacles appeared in all moderate and low ability levels.

In the next stage, each subject formulates arguments, namely establishing relationships and constructing mathematical equations. The initial step in this stage was for subjects to identify and hypothesize unknown information from the problem. Similar to previous studies (Mejía-Ramos, Weber, & Fuller, 2015; Redish & Gupta, 2009; Yu et al., 2017), before forming mathematical equations, it was important for problem solvers to determine unknown information and hypothesize it. The determination of unknown variables aims to assist subjects in the process of solving geometric problems. As revealed in several previous studies, the determination of unknown variables can assist problem solvers in the solving process (Abdillah, 2017; Abdillah et al., 2016; Santia et al., 2019). In the process of formulating arguments, subjects encountered visual representation obstacles, symbolic representation obstacles, and verbal representation obstacles. Visual representation obstacles only appeared in subjects with high abilities. Meanwhile, symbolic representation obstacles and verbal representation obstacles appeared in subjects with moderate and low abilities.

Verification is carried out to provide evidence of calculations, explanations, and interpretations of the calculation process and final results. As previously stated by several researchers, providing evidence of calculation is important for problem solvers to interpret the verification process they carry out (Bakry & Bakar, 2015; Ge & Land, 2003; Jonassen, 1997; Mejíaramos et al., 2015; Pape & Tchoshanov, 2001; Patelli et al., 2014; Prayitno et al., 2020b; Subanji, 2011). These activities show that the subjects understand the procedure for providing evidence based on the problem situation presented (Pape & Tchoshanov, 2001). The verification carried out by the subjects is based on the formulation of previous arguments. In the verification process carried out by the subjects, symbolic representation obstacles and verbal representation obstacles arose at all levels of ability.

Conclusion is the final stage of semantic representation in solving geometric problems. This stage begins with the subject sorting the verification results based on their own methods.

The subjects in this study concluded by calculating the quadratic equation to obtain the value of  $a$ . Next, the subjects calculated the area by substituting it into the square area formula to provide the final answer according to the problem situation. This is in line with previous studies that the final process of problem solving is to provide a conclusion which is the answer to the problem (Mao & Sen, 2018; Mejía-ramos, et al., 2015). The conclusion was given by the subjects to answer the problem provided. In the conclusion process carried out by subjects with high and moderate abilities, symbolic representation obstacles and verbal representation obstacles emerged. Meanwhile, subjects with low abilities only experienced verbal representation obstacles.

### **Semantic Representation Obstacles in the Sorting Stage**

For each topic of the four types, the semantic representation process begins by reading the problem to identify the problem situation presented. In the identification process, subjects sort information about the problem that supports their understanding of the problem situation. In sorting information, the topics are visual, symbolic, and verbal semantic representation types. The purpose of underlining an item is to understand more important information about the problem situation you are reading. This is in line with previous studies (Asay & Schneider, 1976; Riyadi, et al., 2015) that underlining allows students to focus their attention on the problem situation and understand more important information.

Depending on the topic, the sorting stage identifies important and unimportant information about the problematic situation. In addition, each subject identifies missing information about the problem to determine the next process. This activity is in line with the research by Yu, Gan, and Wang (2017) that when dividing text into sentences, the meaning of the problem is not part of the interpretation of the overall problem situation. Sorting is the first step for researchers to understand problematic situations that affect cognitive processes (Lithner, 2008; Panasuk & Beyranevand, 2011; Shi et al., 2015; Subanji, 2011). After identifying the information, the subjects revealed the types of visual, verbal, and symbolic representations and detected the obstacles that occurred. Consistent with previous studies (Anwar & Rahmawati, 2017; Polya, 1973; Vygotsky, 2005), the word order shows that subjects develop an initial picture of the problem they read. The verbal representation of the goal shows the process of thinking and speaking in understanding the problematic situation. At this sorting stage, all types of representation obstacles appear, namely visual representation obstacles, symbolic representation obstacles, and verbal representation obstacles. Visual representation obstacles are in the form of subjects being unable to present in the form of images and subjects being unable to find the length of the sides of triangles and squares. Visual representation obstacles only appear in high-ability subjects (ST). Symbolic representation obstacles appear in all subjects. However, the symbolic representation obstacles that appear differ between subjects. Meanwhile, verbal representation obstacles appear in all subjects. However, these verbal representation obstacles differ in each subject.

### **Semantic Representation Obstacles in the Identification Stage**

After the sorting stage, the next semantic process is keyword identification. Keyword identification refers to students' activity in recognizing, selecting, and interpreting important terms or phrases from the problem situation. This stage is essential because the keywords identified by students become the basis for constructing mathematical relationships in the subsequent stages. As stated by Mao and Sen (2018) and Yu et al. (2017), keyword identification involves identifying, defining, grouping, and interpreting relevant information according to the context of the problem. Therefore, the accuracy of keyword identification influences how students understand the structure of the problem and determine possible solution strategies.

In this study, each subject reread the problem and attempted to identify words or phrases that could guide the solution process. The high-ability subject (ST) identified the keyword

“triangle with a square inside it,” indicating that the subject was able to recognize the main geometric objects in the problem. However, ST still experienced symbolic and verbal representation obstacles because the identified keyword was not immediately connected to the proportional relationship between the triangle and the square. This finding supports the view that recognizing mathematical terms does not automatically ensure that students can construct the intended mathematical meaning from those terms (Mao & Sen, 2018; Yu et al., 2017).

The moderate-ability subject (SS) showed a different pattern. SS identified the word “square” as the main keyword, but the interpretation was limited to the idea that the problem asked for the area of a square. SS did not connect the square to the triangular region in which it was located. This indicates that SS understood the surface meaning of the keyword but failed to interpret its relational meaning in the geometric context. Such difficulty reflects a verbal representation obstacle, because students may be able to mention relevant words but remain unable to explain their mathematical significance in relation to the whole problem situation (Anwar & Rahmawati, 2017; Yu et al., 2017).

A similar verbal representation obstacle was found in the low-ability subject (SR). SR identified “triangle” and “square” as keywords, but these words were treated as separate objects rather than as connected elements in a geometric configuration. SR could not explain how the two shapes were related or how their relationship could support the solution process. This finding shows that keyword identification is not only a linguistic activity, but also a semantic activity that requires students to connect verbal information, mathematical objects, and conceptual relationships (Mao & Sen, 2018; Redish & Gupta, 2009).

Overall, the identification stage shows that representational obstacles may emerge even when students are able to mention relevant keywords. ST experienced symbolic and verbal obstacles, whereas SS and SR mainly experienced verbal obstacles. These findings suggest that students need support not only in recognizing mathematical keywords, but also in interpreting the meaning of those keywords within the structure of the problem. Thus, the identification stage becomes a critical point in the semantic process because failure to interpret keywords accurately can affect the formulation of arguments and the validity of later solution steps (Yu et al., 2017; Mejía-Ramos et al., 2015).

### **Semantic Representation Obstacles in the Argument Formulation Stage**

The argument formulation stage is the stage in which students begin to construct mathematical relationships, determine unknown quantities, and develop equations or strategies to solve the problem. At this stage, students are expected to transform information obtained from the sorting and identification stages into a coherent mathematical argument. This process requires students to determine what is known, what is unknown, and how the known and unknown elements are related. As emphasized by Mejía-Ramos et al. (2015), Redish and Gupta (2009), and Yu et al. (2017), identifying unknown quantities and hypothesizing relationships are important prerequisites before students construct mathematical equations.

In this study, all types of representation obstacles appeared in the argument formulation stage, namely visual, symbolic, and verbal representation obstacles. The high-ability subject (ST) experienced the three types of obstacles simultaneously. ST was able to draw the triangle and square, but the visual representation was not fully accurate because the subject made errors in determining relevant side lengths. This visual obstacle then affected the symbolic process, especially when ST attempted to use similarity to determine the side length of the square. This confirms that visual representations are closely related to symbolic reasoning in mathematical problem solving (Panasuk & Beyranevand, 2011; Gagatsis & Shiakalli, 2004).

ST’s symbolic representation obstacle appeared when the proportional relationship between corresponding sides was not constructed accurately. Although ST understood that similarity was relevant, the mathematical model produced from the visual representation was still incomplete. ST also experienced verbal obstacles because the explanation did not clearly show how the corresponding sides should be compared. This finding indicates that students

may possess relevant conceptual knowledge but still experience difficulty in organizing it into a valid mathematical argument (Mejía-Ramos et al., 2015; Weber, 2001).

The moderate-ability subject (SS) experienced symbolic and verbal obstacles more prominently. SS attempted to formulate an argument by using the formula for area, but the formula was applied without first determining the side length of the square. SS also confused the concepts of area and perimeter, which indicates a symbolic obstacle in selecting and applying mathematical formulas. Verbally, SS could not justify why the selected formula was appropriate, except by referring to the keyword “square.” This shows that students’ arguments may remain superficial when they are based only on keywords rather than on mathematical relationships (Anwar & Rahmawati, 2017; Redish & Gupta, 2009).

The low-ability subject (SR) also experienced symbolic and verbal representation obstacles. SR argued that the perimeter of the triangle should be calculated first and then used to determine the area of the square. This reasoning shows a symbolic misconception because the perimeter of the triangle cannot be directly transformed into the side length or area of the square. SR’s verbal explanation was also based on remembered procedures rather than on the structure of the problem. These findings indicate that the argument formulation stage is highly vulnerable to representational obstacles because students must integrate visual configurations, symbolic models, and verbal justifications into a coherent mathematical argument (Yu et al., 2017; Pape & Tchoshanov, 2001).

### **Semantic Representation Obstacles in the Verification Stage**

Verification is the stage in which students examine the validity of the solution process, provide evidence for their calculations, and interpret whether the obtained results are consistent with the problem situation. Verification is not limited to checking numerical answers; it also involves reviewing the logic of the strategy, the appropriateness of formulas, the accuracy of algebraic operations, and the meaning of the final result. Previous studies emphasize that verification is an important part of mathematical reasoning because it helps students justify and evaluate the solution process they have constructed (Bakry & Bakar, 2015; Ge & Land, 2003; Jonassen, 1997).

In this study, the verification stage showed a different pattern from the previous stages. Visual representation obstacles did not appear in any of the subjects at this stage because the subjects no longer focused on constructing drawings or spatial configurations. Instead, they focused on checking calculations, formulas, and explanations developed in the argument formulation stage. However, symbolic and verbal representation obstacles appeared across all ability levels. This finding indicates that verification mainly requires students to formalize, calculate, and justify the mathematical validity of their solution (Pape, 2004; Subanji, 2012).

The high-ability subject (ST) used algebraic procedures, including a quadratic equation, to determine the side length of the square. Although this strategy was closer to the expected mathematical reasoning, ST still experienced symbolic representation obstacles due to errors in algebraic manipulation. The obstacle was not caused by the absence of relevant concepts, but by inaccuracies in applying symbolic procedures. Verbally, ST also expressed uncertainty about the correctness of the result, showing that the subject could not fully validate the solution. This supports the idea that verification requires both procedural accuracy and meaningful interpretation of mathematical operations (Mejía-Ramos et al., 2015; Prayitno et al., 2020a).

The moderate-ability subject (SS) experienced more serious symbolic and verbal obstacles. SS assumed that the perimeter value obtained could be treated as the area of the square. This indicates a failure to verify whether the calculated quantity matched the quantity required in the problem. The verbal explanation also revealed uncertainty because SS could not justify why perimeter and area could be equated. Such a finding shows that verification fails when students do not distinguish between mathematical quantities or examine the reasonableness of their results (Jonassen, 1997; Ge & Land, 2003).

The low-ability subject (SR) also experienced symbolic and verbal representation obstacles during verification. SR attempted to justify the solution using an incorrect assumption that a triangle is half of a square. This assumption led to an invalid symbolic process and an unsupported verbal explanation. Thus, the verification stage shows that students at all ability levels may struggle to evaluate the validity of their solutions when their symbolic operations and verbal explanations are not grounded in appropriate mathematical relationships. Instruction should therefore emphasize not only obtaining answers, but also checking the coherence between concepts, calculations, and problem contexts (Patelli et al., 2014; Prayitno et al., 2020b).

### **Semantic Representation Obstacles in the Conclusion Stage**

The conclusion stage is the final stage of the semantic representation process in solving geometry problems. At this stage, students are expected to synthesize the results of their previous work, relate them back to the problem statement, and formulate a final answer that is mathematically and contextually appropriate. The conclusion is not merely the last numerical result written by the student, but the product of a complete process of meaning construction. As stated by Mao and Sen (2018) and Mejía-Ramos et al. (2015), the final stage of problem solving requires students to provide an answer that reflects the validity of their reasoning process.

In this study, the conclusion stage began when the subjects attempted to organize the results of their verification and determine whether those results answered the original question. The obstacles that appeared at this stage were different from those in the sorting, identification, argument formulation, and verification stages. The dominant obstacle was verbal representation, although symbolic representation obstacles also appeared in the high- and moderate-ability subjects. This indicates that conclusion-making requires students to connect symbolic results with verbal explanations and contextual meaning (Anwar & Rahmawati, 2017; Pape & Tchoshanov, 2001).

The high-ability subject (ST) was able to state that the final answer should be the area of the square. However, ST still experienced symbolic and verbal representation obstacles because the conclusion was influenced by earlier errors in determining side lengths and conducting algebraic operations. ST's verbal explanation also showed uncertainty about the correctness of the answer. This suggests that even when students understand the form of the expected conclusion, the validity of the conclusion depends on the accuracy of the preceding semantic stages (Mejía-Ramos et al., 2015; Weber, 2001).

The moderate-ability subject (SS) also experienced symbolic and verbal obstacles. SS concluded that the area to be calculated was  $36 \text{ m}^2$ , but this value was obtained by equating perimeter with area. Symbolically, the conclusion was invalid because the mathematical quantity used did not correspond to the quantity required in the problem. Verbally, SS could not confidently justify the answer and indicated doubt about whether the conclusion addressed the question. This shows that students' final conclusions can reflect unresolved misconceptions from earlier stages of problem solving (Jonassen, 1997; Subanji, 2012).

The low-ability subject (SR) primarily experienced verbal representation obstacles. SR concluded that the area of a square could be found using the perimeter of a triangle, but the explanation did not establish a valid mathematical relationship between the two. SR was also unable to provide a confident justification for the final answer. Overall, the conclusion stage demonstrates that students' final answers are strongly influenced by their ability to interpret, connect, verify, and communicate mathematical meaning throughout the semantic process (Mao & Sen, 2018; Yu et al., 2017).

## **CONCLUSION**

This study shows that representational obstacles in students' semantic processes are an important factor affecting their ability to solve geometry problems. These obstacles are not

limited to students' conceptual or procedural understanding, but are also related to difficulties in constructing, interpreting, connecting, and validating mathematical meaning through different forms of representation. The findings indicate that students' use of visual, symbolic, and verbal representations does not automatically lead to correct problem solving when the relationships among these representations are not properly understood.

The patterns of obstacles varied across students' ability levels. The high-ability subject (ST) was generally able to identify relevant information, construct visual representations, and recognize the need to use mathematical concepts such as similarity. However, ST still experienced obstacles in determining accurate side relationships, formulating symbolic models, and verifying algebraic procedures. The moderate-ability subject (SS) showed more dominant symbolic and verbal obstacles, especially in connecting the square to the triangular context and in distinguishing between area and perimeter. Meanwhile, the low-ability subject (SR) experienced fundamental difficulties in explaining the meaning of the problem, establishing valid relationships between geometric elements, and drawing conclusions that were consistent with the problem situation.

Across the five semantic stages, namely sorting, identification, argument formulation, verification, and conclusion, the obstacles appeared in different forms. Visual representation obstacles were more visible in the early stages, particularly when students attempted to construct or interpret geometric drawings. Symbolic representation obstacles became more prominent when students formulated equations, selected formulas, or performed calculations. Verbal representation obstacles appeared across almost all stages, especially when students were required to explain keywords, justify strategies, verify results, and formulate final conclusions. These findings indicate that semantic representation in geometry problem solving is a continuous process in which obstacles at one stage may influence the accuracy of reasoning in the following stages.

Theoretically, this study reinforces the view that students' difficulties in geometry problem solving should be understood not only as errors in concepts or procedures, but also as failures in semantic meaning-making. The study contributes to mathematics education by showing how representational obstacles emerge, shift, and accumulate across stages of problem solving. Practically, the findings suggest that geometry instruction should provide more explicit support for translating among visual, symbolic, and verbal representations. Learning activities should also emphasize mathematical argumentation, interpretation of keywords, construction of valid models, and verification of solution processes so that students can develop stronger semantic understanding in geometry problem solving.

## RECOMMENDATION

Further research is recommended to explore the effectiveness of specific pedagogical interventions, such as the use of dynamic geometry software or metacognitive scaffolding strategies, to mitigate the specific visual and symbolic representation obstacles found at various levels of student ability. In addition, future research could expand the scope of the subject not only to prospective teacher students, but also to students at the secondary education level to map the longitudinal development of semantic obstacles. It is also important to conduct an in-depth analysis of the relationship between individual cognitive styles and the types of representation obstacles that arise in the argument formulation and verification stages.

There are several obstacles or problems that can affect the results of this study, one of which is the complexity of identifying students' internal mental processes, which are often not fully revealed through written documents or clinical interviews alone. Differences in the subjects' initial mathematical ability backgrounds can also be a significant confounding variable if not strictly controlled, given that symbolic and verbal obstacles arise differently at each level of ability. In addition, the limitations of the geometry problem instruments used may

only be able to trigger certain types of obstacles, so that the results of the study may be highly dependent on the characteristics of the problem content presented.

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#### AUTHOR CONTRIBUTIONS STATEMENT

Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
Ririn Dwi Agustin	✓	✓	✓	✓	✓	✓		✓	✓	✓			✓	
Rochsun		✓		✓		✓		✓	✓	✓	✓	✓		✓

#### CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

#### INFORMED CONSENT

We have obtained informed consent from all individuals included in this study.

#### ETHICAL APPROVAL

This research has received permission from the university where the research is being conducted and also received approval from the head of the study program and the subject personally.

#### DATA AVAILABILITY

The data presented is already in the article.

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