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Epistemic Obstacles in Real Analysis: Newman's Error Analysis of Prospective Mathematics Teachers' Theorem Proving

[1]*Ahmad Syamsuadi, [1]Nasrun, [1]Baharullah, [1]Mukhlis, [2]Andi Kaharudin, [3]Joseph Ozigis Akomodi

[1]Mathematics Education, Universitas Muhammadiyah Makassar, Makassar, Indonesia
[2]Mathematics Education, Universitas Lakidende Unaaha, Konawe, Indonesia
[3]Department of Education, Long Island University, New York, United States

*Corresponding Author e-mail: ahmadsyamsuadi@unismuh.ac.id

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Abstract

Real Analysis is often a gatekeeper course in mathematics education, marking the difficult transition from computational calculus to formal axiomatic proof. Prospective mathematics teachers frequently struggle to construct rigorous proofs, even for fundamental properties. This study employs Newman's Error Analysis (NEA) to analyze the epistemological obstacles underlying these struggles. This descriptive qualitative case study involved 15 fifth-semester undergraduates at Universitas Muhammadiyah Makassar who completed a diagnostic proof test on field properties. Analysis of student work revealed that none made errors at the 'Reading' stage. Instead, errors clustered at higher cognitive stages: 3 students exhibited Comprehension errors, 5 made Transformation errors, 4 showed Process Skills errors, and 3 made Encoding errors. From this cohort, four subjects, each exemplifying one of these non-reading error types, were selected for in-depth semi-structured interviews. The findings indicate a hierarchical breakdown in proof construction specifically for the theorem: "If $a \neq 0$ and $a \in \mathbb{R}$, prove that $\frac{1}{a} \neq 0$." Students either failed to translate verbal understanding into explicit premises (Comprehension), could not operationalize an indirect proof strategy (Transformation), neglected to justify algebraic steps with axioms (Process Skills), or omitted formal concluding statements (Encoding). The study concludes that errors often perceived as "carelessness" can be symptoms of deeper epistemological obstacles, such as viewing proof as a computational ritual rather than a formal, communicative argument. These findings underscore the need for explicit instruction on proof mechanics and structure, even when students possess correct mathematical intuition.

Keywords: Error Analysis, Mathematics Education, Newman's Procedure, Real Analysis, Theorem Proving

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INTRODUCTION

The ability to construct logical and rigorous proofs is a cornerstone of advanced mathematics and a primary objective of university-level mathematics education. Unlike elementary calculus, which emphasizes algorithmic computation, Real Analysis requires a fundamental shift toward formal deductive reasoning grounded in axioms and definitions (Andriatna & Dasari, 2025: Aladwan, Alfayez, & Shaheen, 2023; Jäder, Lithner, & Sidenvall, 2020). For prospective mathematics teachers, mastering this domain is critical, not only for strengthening their content knowledge but also for developing the logical foundations necessary to teach future generations. However, Real Analysis is consistently identified as a major stumbling block for undergraduates, often characterized as a "cognitive gap" between intuitive understanding and formal mathematical reasoning (Jones, 2015; Suwarto et al., 2023). The complexity of mathematical proof construction has been extensively documented, with studies revealing that students at various levels struggle with both direct and indirect proofs (Haavold et al., 2024; Quarfoot & Rabin, 2022). These difficulties are often exacerbated by the abstract nature of mathematical objects, which requires students to engage in higher-level thinking and generalization (Andriatna et al., 2025). This struggle is evident globally; international assessments like PISA and TIMSS show that students have greater difficulty with reasoning and justification than with routine calculations, a trend that persists into higher education where learners often rely on intuitive "concept images" rather than formal definitions (Wannapiroon & Pimdee, 2022). This struggle is particularly evident in Real Analysis, where students must navigate the transition from computational thinking to formal axiomatic reasoning (Neuhaus-Eckhardt, Sommerhoff, & Ufer, 2025; Minggi, Arwadi, & Sabri, 2021; Sommerhoff, Kollar, & Ufer, 2021). Similar difficulties are observed in other proof-based courses like linear algebra, where students often fail to grasp the underlying concepts needed for proof construction, even when provided with carefully designed instructional materials (Wirth et al., 2024).

In the Indonesian higher education context, this transition is particularly challenging. The secondary school curriculum has historically emphasized procedural fluency, which can leave students underprepared for the conceptual reasoning and axiomatic rigor demanded by Real Analysis (Aladwan et al., 2023; Aristidou, 2020; Ekasari, 2024; Hodds, Alcock, & Inglis, 2023). This lack of preparedness is further compounded by students' insufficient engagement with self-explanation as a learning strategy, which research has shown to be a powerful tool for improving proof comprehension and conceptual understanding in mathematics courses like calculus and transformation geometry (Atisso, Kachine, & Feyissa, 2023). Consequently, a significant body of literature has documented students' difficulties in proof writing. Frameworks such as syntactic/semantic error analysis (e.g., Stylianides, Stylianides, & Moutsios, 2024) and proof schemes (Harel & Sowder, 2007) have been instrumental in categorizing these challenges. More recently, researchers have developed comprehensive frameworks to understand students' difficulties with specific proof types, such as the Hypothesis Framework for Proof by Contradiction (Quarfoot & Rabin, 2022). Research on proof comprehension has further emphasized that understanding proofs involves both local dimensions (meaning of terms, justification of claims, logical status) and holistic dimensions (modular structure, high-level ideas, transferability) (Davies, Alcock, & Jones, 2020; Haavold et al., 2024). This multi-dimensional view aligns with the resource-based conceptualization of mathematical argumentation skills, which require the availability, coordination, and integration of multiple underlying cognitive resources such as mathematical topic knowledge, methodological knowledge, strategic knowledge, and problem-solving skills (Sommerhoff et al., 2021). Additionally, studies on mathematical abstraction have highlighted how students construct mathematical knowledge through processes of recognition, building-with, and construction (Andriatna et al., 2025). However, while these frameworks often describe *what* errors occur or *how* students conceptualize proof, they may not fully capture the sequential cognitive breakdown during the proving process, from initial problem engagement to final answer formulation.

Newman's Error Analysis (NEA) offers a complementary lens. Originally developed for diagnosing errors in written mathematical problem-solving, NEA provides a hierarchical framework that traces errors through five sequential stages: Reading, Comprehension, Transformation, Process Skills, and Encoding (Newman, 1977; Clements, 1980). This sequential model is valuable because it can pinpoint *where* in the cognitive process a student's reasoning derails. While NEA has been extensively applied to routine problem-solving in arithmetic and algebra (Ekasari, 2024; Noutsara, Neunjhem, & Chemrutsame, 2021), its application to the more complex, multi-step domain of theorem proving in advanced mathematics like Real Analysis remains remarkably limited. Notable exceptions include studies by Kristianto, Mardiyana, & Saputro (2019), who analyzed students' errors in proving convergent sequences using NEA, and Suradi & Djam'an (2021), who investigated errors in group theory proofs based on Newman's framework. These studies found that students commonly make transformation, process skills, and encoding errors when constructing mathematical proofs, highlighting the value of NEA for diagnosing proof-writing difficulties across various mathematical domains. A systematic search of the ERIC, Google Scholar, and Scopus databases using keywords such as "Newman's Error Analysis," "theorem proving," and "Real Analysis" yielded fewer than five studies that explicitly adapt the NEA framework to proof construction tasks. This represents a significant gap, as the hierarchical nature of NEA could provide unique insights into the cognitive hurdles students face when constructing formal, axiomatic proofs.

Therefore, this study addresses this gap by applying Newman's Error Analysis to investigate the specific obstacles students encounter when constructing proofs involving the properties of Real Numbers. The novelty of this approach lies not merely in applying a new framework, but in what NEA can reveal that other frameworks might overlook. For instance, while syntactic error analysis might identify a missing justification as an error, NEA can locate it as a breakdown at the "Process Skills" stage, distinguishing it from an error in transforming the problem into a proof strategy ("Transformation"). Minggi et al. (2021) proposed a structured approach to mathematical proof that includes four key steps: understanding the proposition, deciding the type of proof (direct or indirect), writing the proof in detail, and verifying the validity of the proof. This sequential framework complements NEA by providing a normative model of the proof construction process against which student errors can be mapped. This granular, sequential view can help educators design more targeted interventions. As digital tools become more prevalent in mathematics education, including emerging technologies such as hologram-based teaching (Salmawati et al, 2026; Kaharuddin et al., 2025; Schut et al., 2025), understanding students' foundational cognitive struggles in proof construction becomes even more critical for designing effective instructional interventions. The objectives of this research are to: (1) describe the profile of errors made by undergraduate mathematics students at Universitas Muhammadiyah Makassar in proving a theorem related to the field properties of Real Numbers, categorized according to the NEA framework, and (2) interpret these errors as potential epistemological obstacles, exploring the deeper conceptions of proof that underpin them. Drawing on the theoretical perspectives of mathematical abstraction (Andriatna et al., 2025) and resource-based cognitive skills (Sommerhoff et al., 2021), this study also seeks to understand how students' difficulties reflect underlying challenges in constructing abstract mathematical concepts and coordinating multiple cognitive resources during proof construction. The scope of this study is deliberately focused on a single, foundational theorem the multiplicative inverse property chosen because its standard proof requires an indirect argument (proof by contradiction), a common and conceptually demanding "gatekeeper" task in introductory Real Analysis.

METHOD

Research Design

This study employed a descriptive qualitative case study design. This approach was chosen to gain an in-depth, contextualized understanding of the cognitive processes underlying student errors, moving beyond mere quantification to explore the "why" and "how" of their reasoning (Yin, 2018). The "case" was bounded by a single cohort of students in a specific Real Analysis course at one university, focusing on their engagement with one proof task.

Setting and Participants

The research was conducted at the Faculty of Teacher Training and Education (FKIP), Universitas Muhammadiyah Makassar, within the Mathematics Education Study Programme, during the even semester of the 2023/2024 academic year. The context was the *Real Analysis I* course, a compulsory module covering the Real Number System and its axioms.

Using purposeful sampling, 15 fifth-semester students enrolled in the course were recruited for the study. All participants were provided with information sheets outlining the study's purpose and procedures, and written informed consent was obtained prior to data collection. Anonymity was guaranteed; all student work and interview data were de-identified, with participants assigned pseudonyms (e.g., Subject AS, MR, IN, RI) used in all subsequent analysis and reporting. This study was approved by the institutional ethics committee at Universitas Muhammadiyah Makassar (Approval Number: 123/UNISMUH/2024).

For in-depth analysis, four focal subjects were selected from the initial cohort of 15. The selection criteria were based on their diagnostic test performance and the predominant Newman error type they exhibited (see Table 1).

Table 1. Profile of Selected Focal Subjects

Coded ID	Ability Group (Test Score)	Predominant Newman Error Type	Selection Rationale	Interview Duration
Subject AS	Medium (65)	Comprehension	Exemplar of failing to state premises despite verbal understanding.	25 minutes
Subject MR	Low (40)	Transformation	Exemplar of correctly identifying the goal but being unable to formulate a strategy.	30 minutes
Subject IN	High (85)	Process Skills	Exemplar of correct algebraic manipulation but no axiomatic justification.	22 minutes
Subject RI	High (80)	Encoding	Exemplar of completing the logical derivation but omitting the final conclusion.	18 minutes

Instruments

Two primary instruments were used for data collection:

1. Diagnostic Proof Test: This test consisted of three items designed to assess students' ability to prove fundamental theorems using the field axioms of real numbers.
   - Item 1 (Uniqueness of Additive Identity): Prove that the additive identity in $\mathbb{R}$ is unique.
   - Item 2 (Uniqueness of Multiplicative Inverse): Prove that if $a \neq 0$ and $a \in \mathbb{R}$, then its multiplicative inverse is unique.
   - Item 3 (Focus of this study): Prove that if $a \neq 0$ and $a \in \mathbb{R}$, then $\frac{1}{a} \neq 0$. This item was selected for detailed analysis because its standard proof requires proof by contradiction, demanding a higher level of logical restructuring than direct proofs. This makes it a representative "gatekeeper" task for identifying deeper cognitive obstacles. The selection of this particular theorem is also informed by research on proof by contradiction, which suggests that indirect proofs pose unique challenges for students due to their logical structure and the cognitive demands of reasoning from false assumptions (Quarfoot & Rabin, 2022; Haavold et al., 2024).
2. Semi-Structured Interview Protocol: An interview guide was developed, aligned with Newman's probing procedures. The protocol was designed to prompt students to reconstruct their thinking for each stage of their written solution, helping to distinguish between conceptual errors, procedural slips, and simple carelessness. For example, students were asked, "What were you thinking when you wrote this step?" and "Can you explain why you chose this method?"

Validity and Reliability

Content validity of the diagnostic test and interview protocol was established through expert review by two senior lecturers in Real Analysis and Mathematics Education. They were provided with a validation rubric to assess the items' alignment with course learning outcomes, clarity, and potential to elicit the targeted proof strategies. Based on their qualitative feedback, minor wording adjustments were made to ensure clarity.

For qualitative trustworthiness, this study employed several strategies:

- Triangulation: Methodological triangulation was achieved by comparing data from students' written test responses with their verbatim interview transcripts, ensuring consistency in the interpretation of errors.
- Inter-coder Reliability: Two researchers (the first and second authors) independently coded the written responses of all 15 students using the NEA framework. Initial agreement was 87%. Disagreements were resolved through discussion until a consensus was reached on all codes, strengthening the reliability of the categorical analysis.
- Audit Trail: A detailed record of raw data, coding matrices, and analytical memos was maintained to ensure the transparency and confirmability of the findings.

Data Collection Procedures

Data were collected in three phases:

1. Phase 1 (Diagnostic Test): The 60-minute diagnostic proof test was administered to the 15 participants under standard classroom conditions.
2. Phase 2 (Preliminary Analysis and Subject Selection): The research team analyzed the written responses of all 15 students, coding them according to the NEA framework (see Table 2). Based on this analysis, the four focal subjects representing the core non-reading error types were identified and invited for interviews.
3. Phase 3 (Semi-Structured Interviews): Individual, audio-recorded interviews were conducted with each focal subject within one week of the test. Interviews lasted 18-30 minutes and were transcribed verbatim for analysis.

Data Analysis

Data analysis followed the interactive model of Miles, Huberman, & Saldaña (2020). First, in the data condensation phase, all written responses and interview transcripts were reviewed. Relevant excerpts were selected and coded according to a pre-defined coding scheme based on Newman's Error Analysis, adapted for proof writing. The coding framework is presented in Table 2. This adaptation of NEA for proof writing builds on previous applications of Newman's framework in mathematical proof contexts (Kristianto et al., 2019; Suradi & Djam'an, 2021). Recent studies have successfully combined NEA with other analytical frameworks, such as the SOLO taxonomy, to diagnose student errors in solving first-order differential equations, revealing that errors span from reading to encoding and are often linked to students' low ability in prerequisite concepts like algebra and calculus (Yarman, Murni, & Tasman, 2025). This supports our multi-faceted approach to error analysis. Additionally, the analysis of epistemological obstacles draws on Bachelard & Jones (2002) foundational work on epistemological obstacles, as well as more recent studies on learning obstacles in mathematics education (Munawwaroh et al (2025; Meika et al., 2025). The versatility of Newman's Error Analysis as a diagnostic tool is further demonstrated by its application in special education contexts, where it has been used to identify specific error patterns such as comprehension and encoding errors in blind students learning geometry through Android-based teaching materials (Agustina, Farida, & Irfan, 2024). This underscores the framework's utility across diverse learner populations and instructional formats. Second, for data display, the coded data were organized into matrices and tables to identify patterns and frequencies of error types across the cohort. Finally, conclusion drawing and verification involved interpreting the coded data to construct descriptive profiles of students' proving difficulties. These interpretations were constantly verified by returning to the original data and triangulating between written work and interview statements.

Table 2. Newman's Error Analysis (NEA) Framework Adapted for Proof Writing

NEA Stage	Operational Definition in Proof Context	Example from Student Work
Reading Error	Inability to read/recognize the symbols, terms, or logical structure of the theorem statement.	Misreading "$\frac{1}{a}$" as "a" or not understanding the "if...then" structure.
Comprehension Error	Understands the words but fails to grasp the *logical premises* to be used or the *conclusion* to be shown.	Failing to explicitly state the hypothesis ($a \neq 0$) and the conclusion to be proved ($\frac{1}{a} \neq 0$) at the start of the proof.
Transformation Error	Understands the goal but cannot select or structure an appropriate overall proof strategy (e.g., direct proof, contradiction) or relevant axioms to bridge premise and conclusion.	Knowing the goal is to prove $\frac{1}{a} \neq 0$, and attempting a contradiction but getting stuck after assuming $\frac{1}{a} = 0$, not knowing how to proceed with axioms.
Process Skills Error	Correctly implements the chosen strategy but fails to justify individual steps with the appropriate axioms or logical rules.	Performing algebraic manipulation (e.g., $a \times (\frac{1}{a}) = 1$) without citing the Multiplicative Inverse Axiom.
Encoding Error	Successfully derives the logical result but fails to write the formal concluding statement that links the result back to the original theorem.	Deriving the contradiction 1=0 from the assumption $\frac{1}{a} = 0$, but stopping there without writing "Therefore, $\frac{1}{a} \neq 0$" or "Q.E.D."

RESULTS AND DISCUSSION

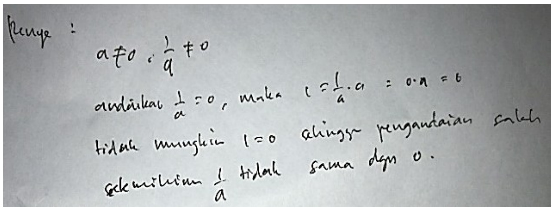
The analysis of the 15 diagnostic tests revealed that none of the students made errors at the 'Reading' stage of the NEA framework. All students could accurately read and restate the theorem. The errors were exclusively located at higher cognitive stages. Table 3 presents the distribution of these errors across the cohort.

Table 3. Distribution of Newman Error Types Across 15 Students

Error Type (NEA Stage)	Number of Students (n=15)	Percentage	Focal Subject(s) Exemplifying the Error
Reading	0	0%	-
Comprehension	3	20%	AS
Transformation	5	33.3%	MR
Process Skills	4	26.7%	IN
Encoding	3	20%	RI

The distribution of error types observed in this study aligns with findings from other investigations of proof construction errors. Kristianto et al. (2019) reported similar patterns in students' proofs of convergent sequences, noting that high-ability students tended to make process skill errors, while moderate and low-ability students exhibited transformation and encoding errors. Suradi & Djam'an (2021) found that in group theory proofs, students predominantly struggled with transformation and encoding stages, with 100% of participants making errors at these levels. The absence of reading errors in our study is also consistent with these previous findings, suggesting that students at this level generally possess adequate reading comprehension but struggle with higher-order cognitive processes involved in proof construction. The following sections provide a detailed qualitative account of each error type as exhibited by the four focal subjects, drawing on their written work and interview excerpts. The results are presented descriptively; interpretation and linkage to theory are reserved for the Discussion section.

Subject AS's written response is shown in Figure 1. The script begins immediately with algebraic manipulation ($a \bullet \frac{1}{a} = 1$) and does not contain any explicit statement of the hypothesis ($a \neq 0$) or the conclusion to be proved ($\frac{1}{a} \neq 0$). There is no setup or logical framing of the argument.



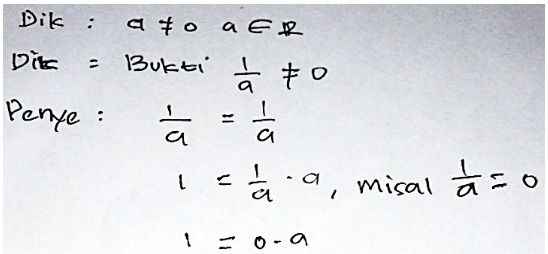
Solution:
$a \neq 0, \frac{1}{a} \neq 0$
Assume a = 0, then $c = \frac{1}{a}$, c = 0, q = 0
It is impossible for c = 0, so the assumption is wrong. Therefore, 1/a is not equal to 0.

Figure 1. A typed transcription of Subject AS's written response would go here. It would show a line of algebraic work, with a caption like: "Transcription of Subject AS's response, showing the absence of stated premises and conclusion.

In the interview, Subject AS demonstrated a clear verbal understanding of the task's requirements:

Researcher: "Can you explain what is known and what is being asked in this theorem?"
Subject AS: "Yes, it is known that a is a real number and not equal to zero. And it is asked to prove that 1 divided by a is also not equal to zero."
Researcher: "Why didn't you write that down at the beginning of your proof?"
Subject AS: "I forgot to write it down... I thought I could just start solving it directly."

Subject MR's written response (Figure 2) correctly sets up a proof by contradiction. The student writes "Andaikan $\frac{1}{a} \neq 0$ (Assume $\frac{1}{a} \neq 0$). However, the proof then stalls. The student attempts to multiply both sides by a but does not apply the field axioms to derive the contradiction formally, leaving the argument incomplete.



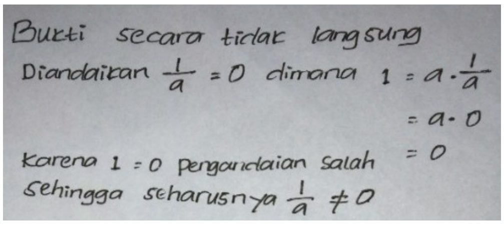
Given: $a \neq 0, a \in \mathbb{R}$
To prove: $\frac{1}{a} \neq 0$
Solution:
$\frac{1}{a} = \frac{1}{a}$
$1 = (\frac{1}{a}) \bullet a$, suppose $\frac{1}{a} = 0$
$1 = 0 \bullet a$

Figure 2. A typed transcription of Subject MR's response would go here. It would show the initial assumption, the stalled manipulation, with a caption like: "Transcription of Subject MR's response, showing a stalled contradiction proof after the initial assumption.

The interview excerpt confirms the student's inability to transform the assumption into a valid proof structure:

Subject MR: "I assumed $\frac{1}{a} = 0$, then I multiplied by a to get 1 = 0. But I felt confused, sir... It's impossible for 1 to equal 0, so I didn't know what to do next. I thought I must have made a mistake."
Researcher: "So you felt you couldn't continue because you arrived at something impossible?"
Subject MR: "Yes, exactly. If it's impossible, my assumption must be wrong... but is that allowed? I didn't know how to write it as a proof."

Subject IN's written proof (Figure 3) is logically and algebraically sound. The student assumes $\frac{1}{a} = 0$, multiplies both sides by a, and correctly concludes 1 = 0, a contradiction. However, the proof is presented as a series of algebraic manipulations without any explicit justification for the key step: multiplying both sides by a. The foundational axioms (e.g., Multiplicative Inverse Axiom to justify $a \cdot \frac{1}{a} = 1$, or the Multiplicative Property of Equality) are not cited.



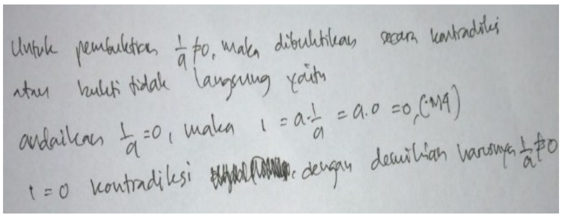
Indirect proof:
Assume $\frac{1}{a} = 0$, where $1 = a \cdot (\frac{1}{a})$
$= a \cdot 0$
Since $1 = 0$, the assumption is false,
therefore $\frac{1}{a} \neq 0$

Figure 3. A typed transcription of Subject IN's response would go here. It would show the clean algebraic derivation, with a caption like: "Transcription of Subject IN's response, showing correct algebra but a complete absence of axiomatic justification

When asked about the justification for the step, Subject IN revealed a procedural, rather than axiomatic, understanding:

Researcher: "You multiplied both sides by a and got 1 = 0. Can you tell me which property of real numbers allows you to do that step?"
Subject IN: "I just did the algebra... it's the inverse, right? a times $\frac{1}{a}$ is 1. I forgot, sir, that there is a specific axiom for that, like the M4 field characteristic. I just know it's true."

Subject RI's written response (Figure 4) successfully navigates the logical steps of the proof by contradiction. The student assumes $\frac{1}{a} = 0$, correctly manipulates the equation using the field axioms to derive the contradiction $1 = 0$. However, the answer sheet ends abruptly at the derived contradiction. There is no concluding statement, such as "Therefore, our assumption is false and $\frac{1}{a} \neq 0$," or "Q.E.D."



Assume $\frac{1}{a} = 0$, then $1 = a \cdot \frac{1}{a} = a \cdot 0 = 0$. (contradiction)
Since $1 = 0$, a contradiction, therefore $\frac{1}{a} \neq 0$.

Figure 4. A typed transcription of Subject RI's response would go here. It would show the full derivation ending with "1 = 0", with a caption like: "Transcription of Subject RI's response, showing the derivation but the absence of a final concluding statement."

The interview reveals that the student considered the proof complete without a formal conclusion:

Researcher: "Your derivation is correct. Is the proof finished here [pointing to the final line '1 = 0']?"
Subject RI: "Yes, it's proven. The contradiction is there, so it's done."
Researcher: "Why didn't you write a sentence or 'Q.E.D.' at the end to state that the theorem is proved?"
Subject RI: "I just forgot to write it. I thought it was obvious from the work.

This study aimed to investigate the epistemological obstacles underlying prospective teachers' difficulties in proving a foundational Real Analysis theorem. By applying Newman's Error Analysis (NEA) to the proof task, we were able to pinpoint breakdowns at four distinct cognitive stages, each of which points to a deeper epistemological obstacle than mere content knowledge deficits.

The absence of Reading Errors across the cohort suggests that the primary barrier for these students was not decoding the language or symbols of the theorem, but engaging with its logical and structural demands. This finding aligns with the work of Hodds et al. (2023), who argue that students often struggle with the epistemic norms of proof the unspoken rules of what constitutes a valid argument rather than the mathematical objects themselves.

The Comprehension Error exhibited by Subject AS reveals what we term a meta-comprehension obstacle. The student understood the mathematical content but failed to appreciate the *epistemic norm* of proof-writing. The student treated the proof as a calculation task where "showing work" regarding the premises was optional, rather than a logical argument where premises must be explicit. This finding is consistent with research by Harel & Sowder (2007) on proof schemes. Subject AS appeared to operate with an *external conviction* proof scheme, where the goal is to perform a ritual (like algebraic manipulation) to produce an answer, rather than an *axiomatic* scheme, where the goal is to build a logical structure from accepted truths. The error here is not a lack of knowledge, but a failure to value the structural requirements of Real Analysis, suggesting that for this student, proving is synonymous with solving.

Subject MR's Transformation Error points to an ontological obstacle inherent in indirect proof. The student intuitively knew to use a contradiction approach but was paralyzed upon reaching the "impossible" statement 1=0. As Lenzen (2018) and Antonini & Mariotti (2008) have noted, proof by contradiction requires a sophisticated cognitive leap: temporarily inhabiting a logical world where a falsehood is assumed true and using that falsehood to derive a contradiction, thereby validating the original claim. Quarfoot & Rabin (2022) further elaborated on this phenomenon through their 'False World Hypothesis,' which suggests that students struggle with proof by contradiction because they find it difficult to reason within a counterfactual world where false assumptions are temporarily treated as true. However, their empirical study with introduction-to-proof students found limited evidence for this hypothesis, suggesting instead that resource-related issues and difficulties with quantifier negation may be more significant sources of student difficulties with indirect proofs. Minggi, Arwadi, & Sabri (2021) addressed this challenge by proposing a structured approach to proof by contradiction that includes explicit scaffolding through proof flowcharts and visual diagrams, demonstrating that when students are guided through systematic steps, they become better equipped to handle the cognitive demands of indirect proofs. MR's confusion ("I thought I must have made a mistake") indicates a failure to accept the logic of contradiction that deriving an impossibility is the *goal*, not a mistake. The student struggled to sustain the suspension of truth long enough to complete the logical circle, viewing the derivation of 1=0 as evidence of a procedural error rather than a successful proof strategy. This finding resonates with the resource-based conceptualization of mathematical argumentation skills proposed by Sommerhoff, Kollar, & Ufer (2021). They identified mathematical strategic knowledge as one of four key resources underlying proof construction abilities, alongside mathematical topic knowledge, methodological knowledge, and problem-solving skills. Subject MR's difficulty exemplifies how deficiencies in strategic knowledge the ability to link specific cues in mathematical tasks with appropriate methods and concepts can impede the proof construction process, even when students possess adequate topic knowledge.

The Process Skills Error of Subject IN highlights a ritualistic behavior rooted in a procedural-justification deficit. The student possessed the procedural fluency to manipulate symbols correctly ($a \cdot \frac{1}{a} = 1$) but lacked the declarative knowledge to ground this step in the axiomatic structure of real numbers. Subject IN treated the algebraic cancellation as a self-evident fact learned by rote, whereas Real Analysis requires viewing it as an instantiation of the Multiplicative Inverse Axiom (M4). This supports observations by Miller & Hudson (2007) and Suhady, Roza, & Maimunah (2023) that students trained in procedural algebra struggle to adapt to the declarative nature of advanced mathematics. The missing justification is not merely a lapse in memory, but evidence that the student views axioms as invisible background noise rather than the essential scaffolding of the proof. The distinction between procedural fluency and declarative knowledge observed in Subject IN aligns with broader research on mathematical abstraction. Andriatna et al. (2025) note that abstraction involves both empirical abstraction (based on concrete experiences) and reflective abstraction (involving the reorganization of existing knowledge structures). Subject IN's automatic performance of algebraic cancellation without axiomatic justification represents empirical abstraction without the reflective abstraction needed to understand the underlying mathematical structure. This finding underscores the importance of designing learning experiences that promote the transition from empirical to reflective abstraction, as emphasized in recent systematic reviews of mathematical abstraction research. The student focuses on the *result* of the operation, not the *validity* of the step. Emerging technologies, such as interactive theorem provers (ITPs), offer a potential bridge between procedural and declarative knowledge. As explored by Iannone & Thoma (2024), students learning to use the ITP Lean reported that the software's demand for rigorous, step-by-step justification forced them to confront the 'why' behind their algebraic manipulations, thereby strengthening their understanding of foundational axioms. However, their study also highlights a critical challenge: many students perceive the difficult syntax of ITPs as a barrier disconnected from 'real' mathematics, leading to disengagement. This suggests that while ITPs hold promise for addressing the 'ritualistic behavior' we observed, their implementation requires careful pedagogical scaffolding to help students see the connection between programming a proof and constructing a formal mathematical argument (Iannone & Thoma, 2024).

Finally, Subject RI's Encoding Error reveals a communicative obstacle in the conception of proof. The student successfully engaged in the cognitive labor of the proof the logical derivation but failed to recognize that a proof is a communicative act requiring formal closure. By omitting the concluding statement, Subject RI revealed a 'private' view of proof, where the goal is to convince oneself (Gila & Hans, 2010). The student assumed that since the derivation (1=0) was visible, the conclusion was self-evident. This communicative obstacle reflects what Haavold, Roksvold, & Sriraman (2024) describe as the gap between students' understanding of proof as a personal cognitive activity versus its role as a public, communicative act within the mathematical community. Their research on pre-service teachers' knowledge of and beliefs about direct and indirect proofs reveals that many students hold narrow and rigid views about what constitutes a valid mathematical argument, often prioritizing personal conviction over formal logical structure. This contrasts with the 'public' view of proof required in academic mathematics, where the goal is to construct a complete, transparent argument for a reader. This "carelessness" is therefore a symptom of a deeper issue: the student has not internalized the full, communicative structure of a deductive argument (hypothesis → argument → conclusion).

While these findings are consistent with previous research on proof difficulties, the NEA framework adds value by providing a diagnostic map of *where* in the proving process these epistemological obstacles surface. This sequential view has implications for instruction. For instance, a student like IN might have been mistakenly praised for "getting the right idea," whereas NEA identifies a critical gap in Process Skills that requires targeted intervention on axiomatic justification. The resource-based approach to mathematical argumentation (Sommerhoff et al., 2021) provides a complementary perspective on how to address such gaps. Their comparison of sequential versus concurrent instructional approaches for supporting proof construction skills revealed that while both approaches can effectively develop underlying resources, mathematical strategic knowledge benefits significantly more from concurrent instruction that integrates multiple resources simultaneously. This suggests that interventions for students like MR and IN might be most effective when they combine explicit attention to strategic knowledge with opportunities to apply this knowledge in authentic proof construction contexts.

This study has several limitations. First, the findings are based on a single proof task in a single cohort, which limits the generalizability of the claims. The specific nature of an indirect proof may have amplified certain obstacles, like the Transformation Error. Second, the interpretation of these errors as "epistemological obstacles" is one of several possible interpretations. Alternative factors, such as assessment pressure, time constraints during the test, or students' representational fluency, could also have contributed to the observed errors. Future research could employ a longitudinal design or a broader set of proof tasks to see if these obstacles persist across different types of proofs and contexts. The systematic literature review by Andriatna, Nurhasanah, & Shahrill (2025) on mathematical abstraction research from 2016-2022 reveals that most studies in this domain employ qualitative methods at the junior high school level, focusing on geometry topics. This highlights the need for more research at the university level, particularly in advanced mathematical domains like Real Analysis, to understand how abstraction processes evolve as students encounter increasingly formal mathematical concepts. Additionally, the integration of NEA with theoretical frameworks from abstraction research and resource-based skill development could provide a more comprehensive understanding of the cognitive processes underlying proof construction difficulties.

CONCLUSION

This exploratory study investigated the proof-writing difficulties of prospective mathematics teachers by applying Newman's Error Analysis to a foundational Real Analysis theorem. The findings move beyond identifying simple errors, suggesting that student failures are not merely the result of weak content knowledge, but are often symptoms of deeper, underlying epistemological obstacles. These obstacles meta-comprehension, ontological, ritualistic, and communicative manifest at distinct cognitive stages of the proving process. The study's contribution lies in mapping these obstacles using NEA, offering a new lens to view phenomena often dismissed as "carelessness." It reveals a critical disconnection: students may possess the procedural ability to manipulate symbols but lack the syntactic and epistemic understanding of what constitutes a formal mathematical argument. The study is exploratory and its claims are limited to the specific task and context. However, it provides a foundation for future research and suggests practical implications for teaching.

RECOMMENDATION

Based on the findings, the following actionable teaching interventions are recommended, linked directly to each identified obstacle:

- Addressing Meta-Comprehension Obstacles: The curriculum for Real Analysis should begin with explicit modules on "Proof Writing Mechanics" that teach the structural grammar of a proof, including the mandatory declaration of premises and the conclusion, using templates and sentence starters.
- Addressing Ontological Obstacles: To build students' comfort with indirect proof, lecturers should incorporate "error-based learning" sessions. Students can critique incomplete or stalled proofs (like Subject MR's) and discuss the logical validity of reasoning from a false assumption. This can help normalize the cognitive conflict of proof by contradiction.
- Addressing Ritualistic Behavior: Instruction must explicitly connect algebraic procedures to the underlying axioms. A "justification audit" can be used where students are required to write the axiom or theorem used to justify each step in their proof, bridging the gap between procedural fluency and declarative knowledge.
- Addressing Communicative Obstacles: The social and communicative function of proof should be emphasized. Students can engage in peer-review activities where they read and critique each other's proofs, focusing not just on logical correctness but on the completeness and clarity of the communication, including the final concluding statement.

Future research should expand this analysis by applying the NEA framework to a wider range of proof tasks across multiple universities to determine the generalizability of these epistemological obstacle patterns and to develop and test the effectiveness of these targeted instructional interventions.

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AUTHOR CONTRIBUTIONS STATEMENT

Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
Ahmad Syamsuadi	✓	✓		✓	✓	✓		✓	✓	✓			✓	
Nasrun		✓				✓		✓	✓	✓	✓	✓		
Baharullah	✓		✓				✓	✓			✓		✓	
Mukhlis		✓			✓	✓		✓			✓	✓		
Andi Kaharudin			✓	✓			✓	✓		✓		✓		
Joseph Ozigis Akomodi				✓			✓							

CONFLICT OF INTEREST STATEMENT

The authors declare that there is no conflict of interest.

INFORMED CONSENT

We have obtained informed consent from all individuals included in this study.

ETHICAL APPROVAL

This study was conducted in accordance with all applicable national regulations and institutional policies. All procedures involving human participants adhered to the ethical principles outlined in the Declaration of Helsinki.

DATA AVAILABILITY

The data that support the findings of this study can be obtained from the corresponding author upon reasonable request.

REFERENCES

Agustina, R., Farida, N., & Irfan, M. (2024). Learning mathematics outcomes using Android for blind students based on Newman's theory. *Journal of Education and Learning, 18*(3), 1056–1065.

Aladwan, S., Alfayez, M., & Shaheen, H. (2023). The level of conceptual and procedural knowledge in mathematics and its relationship to creative thinking among gifted students. *International Journal of Instruction, 16*(4), 897–916. https://doi.org/10.29333/iji.2023.1648a

Andriatna, R., & Dasari, D. (2025). Sentiment analysis of pre-service mathematics teacher through naïve Bayes classifier: The case of mathematical abstraction problem. *Barekeng: Jurnal Ilmu Matematika dan Terapan, 19*(3), 1485–1498. https://doi.org/10.30598/barekengvol19iss3pp1485-1498

Andriatna, R., Nurhasanah, F., & Shahrill, M. (2025). Understanding mathematical abstraction: A systematic literature review of its conceptualizations and research practices. *Infinity Journal, 14*(4), 1081–1104. https://doi.org/10.22460/infinity.v14i4.p1081-1104

Antonini, S., & Mariotti, M. A. (2008). Indirect proof: What is specific to this way of proving? *ZDM Mathematics Education, 40*(3), 401–412. https://doi.org/10.1007/s11858-008-0091-2

Aristidou, B. M. (2020). Is mathematical logic really necessary in teaching mathematical proofs? *Athens Journal of Education, 7*(1), 99–116. https://doi.org/10.30958/aje.7-1-5

Atisso, A., Kachine, D., & Feyissa, D. (2023). Effects of self-explanation on the expansion and indulgent of mathematics teaching: Case of the students of the University of Wachemo. *Qubahan Academic Journal, 3*(1), 1–12. https://doi.org/10.48161/qaj.v3n1a128

Bachelard, G. (2002). *The formation of the scientific mind: A contribution to a psychoanalysis of objective knowledge* (M. M. Jones, Trans.).

Clements, M. A. (1980). Analyzing children's errors on written mathematical tasks. *Educational Studies in Mathematics, 11*(1), 1–21. https://doi.org/10.1007/BF00369157

Davies, B., Alcock, L., & Jones, I. (2020). Comparative judgement, proof summaries and proof comprehension. *Educational Studies in Mathematics, 105*(2), 181–197. https://doi.org/10.1007/s10649-020-09984-x

Ekasari, A. (2024). Error analysis of learners' problem-solving abilities in mathematics courses: A Newman Error Analysis (NEA) approach. *Jurnal Pendidikan Sains, 12*(1), 45–54. https://doi.org/10.17977/2442-3904.1019

Gila, H., & Hans, N. J. (Eds.). (2010). *Explanation and proof in mathematics: Philosophical and educational perspectives*. Springer.

Haavold, P., Roksvold, J., & Sriraman, B. (2024). Pre-service teachers' knowledge of and beliefs about direct and indirect proofs. *Journal of Mathematical Behavior, 16*(4), 336–355. https://doi.org/10.1080/19477503.2024.2363710

Harel, G., & Sowder, L. (2007). Toward comprehensive perspectives on the learning and teaching of proof. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 805–842). Information Age Publishing.

Hodds, M., Alcock, L., & Inglis, M. (2023). Self-explanation training improves proof comprehension. *Journal for Research in Mathematics Education, 54*(2), 112–131. https://doi.org/10.5951/jresematheduc.45.1.0062

Iannone, P., & Thoma, A. (2024). Interactive theorem provers for university mathematics: An exploratory study of students' perceptions. *International Journal of Mathematical Education in Science and Technology, 55*(10), 2622–2644. https://doi.org/10.1080/0020739X.2023.2178981

Jäder, J., Lithner, J., & Sidenvall, J. (2020). Mathematical problem solving in textbooks from twelve countries. *International Journal of Mathematical Education in Science and Technology, 51*(7), 1120–1136. https://doi.org/10.1080/0020739X.2019.1656826

Jones, S. R. (2015). Calculus limits involving infinity: The role of students' informal dynamic reasoning. *International Journal of Mathematical Education in Science and Technology, 46*(1), 105–126. https://doi.org/10.1080/0020739X.2014.941427

Kaharuddin, A., García, J. G., Magfirah, I., & Yulismayanti, Y. (2025). Validating a TPCK-S instrument for hologram-based mathematics teaching. *Indonesian Journal on Learning and Advanced Education (IJOLAE), 7*(3), 525–536. https://doi.org/10.23917/ijolae.v7i3.12119

Kristianto, E., Mardiyana, & Saputro, D. R. S. (2019). Analysis of students' error in proving convergent sequence using Newman error analysis procedure. *Journal of Physics: Conference Series, 1180*, Article 012001. https://doi.org/10.1088/1742-6596/1180/1/012001

Lenzen, W. (2018). Leibniz's ontological proof of the existence of God and the problem of impossible objects. In *Leibniz's general critique of the ontological proof* (pp. 1–21). Springer. https://doi.org/10.1007/s11787-017-0159-2

Meika, I., Sartika, N. S., Sujana, A., Jarinah, Hakim, Z., Windiarti, I. S., & Hendra. (2025). E-didactics design of differential calculus based on TPACK to overcome learning obstacles for mathematics pre-service teachers. *Infinity Journal, 14*(3), 733–752. https://doi.org/10.22460/infinity.v14i3.p733-752

Miles, M. B., Huberman, A. M., & Saldaña, J. (2020). *Qualitative data analysis: A methods sourcebook* (4th ed.). SAGE Publications.

Miller, S. P., & Hudson, P. J. (2007). Using evidence-based practices to build mathematics competence related to conceptual, procedural, and declarative knowledge. *Learning Disabilities Research & Practice, 22*(1), 47–57. https://doi.org/10.1111/j.1540-5826.2007.00230.x

Minggi, I., Arwadi, F., & Sabri. (2021). The mathematical proof steps of mathematics study program students in the subject of real analysis. *Journal of Physics: Conference Series, 1899*, Article 012146. https://doi.org/10.1088/1742-6596/1899/1/012146

Munawwaroh, A. G., Pramudya, I., & Nurhasanah, F. (2025). Epistemological obstacles in the process of learning mathematical abstraction: A systematic literature review. *KnE Social Sciences, 10*(11), 272–294. https://doi.org/10.18502/kss.v10i11.18749

Neuhaus-Eckhardt, S., Sommerhoff, D., & Ufer, S. (2025). Proof comprehension, proof validation, proof construction: All the same or different skills? An answer based on an empirical analysis. *International Journal of Science and Mathematics Education*. Advance online publication. https://doi.org/10.1007/s10763-025-10621-3

Newman, M. A. (1977). An analysis of sixth-grade pupils’ errors on written mathematical tasks. *Victorian Institute for Educational Research Bulletin, 39*, 31–43.

Noutsara, S., Neunjhem, T., & Chemrutsame, W. (2021). Mistakes in mathematics problems solving based on Newman’s error analysis on set materials. *Journal La Edusci, 2*(1), 15–22. https://doi.org/10.37899/journallaedusci.v2i1.367

Quarfoot, D., & Rabin, J. M. (2022). A hypothesis framework for students’ difficulties with proof by contradiction. *International Journal of Research in Undergraduate Mathematics Education, 8*, 490–520. https://doi.org/10.1007/s40753-021-00150-z

Rabin, J. M., & Quarfoot, D. (2022). Sources of students’ difficulties with proof by contradiction. *International Journal of Research in Undergraduate Mathematics Education, 8*(3), 521–549. https://doi.org/10.1007/s40753-021-00152-x

Salmawati, Kaharuddin, A., Syam, N., & Gupta, V. (2026). Teacher technological pedagogical content knowledge and skills (TPCK-S) and student cognitive agency: Determinants of deep learning outcomes in immersive geometry instruction. *Jurnal Kependidikan: Jurnal Hasil Penelitian dan Kajian Kepustakaan di Bidang Pendidikan, Pengajaran, dan Pembelajaran, 12*(1). https://doi.org/10.33394/jk.v12i1.19264

Schut, L., Tomašev, N., McGrath, T., Hassabis, D., Paquet, U., & Kim, B. (2025). Bridging the human-AI knowledge gap through concept discovery and transfer in AlphaZero. *Proceedings of the National Academy of Sciences, 122*(13), Article e2406675122. https://doi.org/10.1073/pnas.2406675122

Sommerhoff, D., Kollar, I., & Ufer, S. (2021). Supporting mathematical argumentation and proof skills: Comparing the effectiveness of a sequential and a concurrent instructional approach to support resource-based cognitive skills. *Frontiers in Psychology, 11*, Article 572165. https://doi.org/10.3389/fpsyg.2020.572165

Stylianides, G. J., Stylianides, A. J., & Moutsios, A. (2024). Proof and proving in school and university mathematics education research: A systematic review. *ZDM - Mathematics Education, 56*(1), 47–59. https://doi.org/10.1007/s11858-023-01518-y

Suhady, W., Roza, Y., & Maimunah, M. (2023). Identification of students’ conceptual and procedural errors in solving problems in three-dimensional material. *Devotion: Journal of Research and Community Service, 4*(3), 788–797. https://doi.org/10.36418/devotion.v4i3.433

Suradi, & Djam’an, N. (2021). Students’ error on proof of the group with “satisfy axioms proof” based on Newman error analysis. *Journal of Physics: Conference Series, 2123*, Article 012024. https://doi.org/10.1088/1742-6596/2123/1/012024

Suwarto, S., Hidayah, I., Rochmad, R., & Masrukan, M. (2023). Intuitive thinking: Perspectives on intuitive thinking processes in mathematical problem solving through a literature review. *Cogent Education, 10*(2), Article 2243119. https://doi.org/10.1080/2331186X.2023.2243119

Wannapiroon, N., & Pimdee, P. (2022). Thai undergraduate science, technology, engineering, arts, and math (STEAM) creative thinking and innovation skill development: A conceptual model using a digital virtual classroom learning environment. *Education and Information Technologies, 27*, 5689–5716. https://doi.org/10.1007/s10639-021-10849-w

Wirth, L., Kirsten, K., Serpe, C., & Greetham, G. (2024). Comparing different types of instructional videos in a flipped proof-based classroom. *Frontiers in Education, 9*, Article 1438355. https://doi.org/10.3389/feduc.2024.1438355

Yarman, Y., Murni, D., & Tasman, F. (2025). Implementation of SOLO taxonomy and Newman error analysis in first-order differential equation. *Infinity Journal, 14*(3), 695–710. https://doi.org/10.22460/infinity.v14i3.p695-710

Yin, R. K. (2018). *Case study research and applications: Design and methods* (6th ed.). SAGE Publications.