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Tracing Students' Cognitive Progression in Geometry: A Design-Based Research Through the Van Hiele Model of Instruction and Ethnomathematics

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Abstract

The 21st century demands that learners not only master knowledge, but also critical, creative, collaborative, and communicative thinking skills in response to the global challenges of the industrial revolution era 4.0. However, these skills have not been optimally developed in science learning because learning practices remain teacher-centered and do not provide space for high-level thinking activities that address the low abilities of the 21st century. On the other hand, increasing environmental, social, and economic problems underscore the need for education that fosters sustainability awareness. Education for Sustainable Development (ESD) is a potential approach to integrating 21st-century skills and sustainability values in science learning. However, the implementation of ESD in Indonesia remains limited. It has not been entirely directed toward strengthening 21st-century skills, so a more in-depth study is needed to assess the extent to which ESD has been applied in science learning and its contribution to the development of these competencies. This study aims to analyze the implementation of ESD in science learning to develop 21st-century skills in Indonesia through a Systematic Literature Review of 25 published articles from Google Scholar, Semantic Scholar, and ResearchGate, published between 2020 and August 26, 2025. The article selection process followed the inclusion and exclusion criteria and guidelines of PRISMA 2020, resulting in 25 studies deemed appropriate and relevant. The results of the analysis show that the application of ESD in science learning focuses on developing tools aligned with the learning models used, including Problem-Based Learning (PBL), Project-Based Learning (PjBL), Predict–Observe-Explain (POE), RADEC, and SSCS. The PBL model is the most dominant one because it effectively encourages students to analyze and solve environmental problems scientifically. In addition, most of the research uses e-modules, ESD-based LKPDs, digital media, and virtual field trips as learning media. Based on the analysis of 21st-century skills, critical thinking is the most improved competency (48.3%), followed by sustainability awareness and problem-solving (13.8%), creative thinking and science literacy (10.3%), and environmental literacy (3.4%). ESD integration has been proven to strengthen students' ability to analyze phenomena, evaluate solutions, and make sustainability-oriented decisions. The material topics most often associated with ESD include Environment and Sustainability (44.8%) and Energy and Force (34.5%), reflecting the relevance of climate change, ecosystem, and renewable energy issues in science learning.

Keywords: cognitive progression; design-based research; ethnomathematics; geometry; Van Hiele model of instruction

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INTRODUCTION

Geometry is a fundamental component of the school mathematics curriculum. It plays a strategic role in developing the foundations of mathematical thinking, extending beyond practical applications in daily life to serve as a prerequisite for mastering more advanced mathematical topics (Gridos et al., 2022; Hwang et al., 2020; Patahuddin et al., 2020). According to Haj-Yahya (2023), geometry is closely connected to everyday life and acts as a bridge for the development of higher-order mathematical thinking. Similarly, Zhang et al. (2025) and Fujita et al. (2020) assert that geometry is the mathematical domain most visibly related to the physical world, enabling the visualization of concepts from other branches of mathematics and providing tangible examples of the diverse structures within mathematical systems.

For decades, geometry has been viewed not merely as a collection of concepts and theorems, but as a medium for cultivating logical reasoning, spatial thinking, and deep problem-solving skills. Nevertheless, geometry remains one of the most challenging topics for many students. At the lower secondary level, students often struggle to comprehend the hierarchical relationships among quadrilaterals, prove their properties, and apply geometric reasoning to solve problems (Gusfitri et al., 2022; Kusaeri et al., 2024; Nathan et al., 2021; Özçakır & Çakıroğlu, 2022). Research by Spiller et al. (2023) reported that students' average achievement in geometry reached only 41.84%, lower than in algebra, number theory, and probability. They found that students had difficulty distinguishing the properties of quadrilaterals and identifying interrelationships among them. Similarly, Chen et al. (2021) revealed that most students scored below the minimum mastery criterion. These findings indicate a persistent gap between the intended learning goals of geometry and students' actual learning outcomes.

Within the national curriculum, geometry constitutes approximately 42% of the total mathematics content. This proportion reflects the inherent complexity of geometry learning compared to other mathematical strands. Such complexity arises from its broad scope, which encompasses the recognition of geometric objects (points, lines, planes, and spaces), the relationships among these objects, and algebraic representations of geometric problems. Given this complexity, students' learning experiences in geometry should not be fragmented but rather structured progressively—from concrete experiences (direct interaction with geometric objects) to abstract reasoning (through intuition, imagination, and interconnectivity) (Harris et al., 2021; Michael–Chrysanthou et al., 2024; Spiller et al., 2023). Bernabeu et al. (2024) and Arnal-Bailera & Manero (2024) emphasized that students' ability to comprehend and construct geometric concepts is strongly influenced by their geometric thinking levels, as articulated in the Van Hiele Model of Instruction**.**

Research on the Van Hiele model has been widely conducted to understand students' development in geometric thinking (Rahmayani et al., 2024; Santos et al., 2022). Arnal-Bailera & Manero (2024) developed the characteristics of Van Hiele's level 5 using the Delphi method, emphasizing the importance of learning activities that target the highest level to strengthen mastery of the previous levels. Meanwhile, Mudhefi et al., (2024) used the Van Hiele model to identify students' learning difficulties in Euclidean geometry, revealing that many students remain at lower levels of geometric thinking. On the other hand, ethnomathematics has been recognized as an effective approach to connect mathematical concepts with local cultural contexts. Batiibwe, (2024) highlighted how cultural practices such as traditional games and weaving can be used to enhance students' conceptual understanding and critical thinking skills. Tamur et al., (2023) also showed that research on ethnomathematics is growing globally, yet its integration with cognitive learning theories like the Van Hiele model remains limited. Furthermore, the Design-Based Research (DBR) approach has been widely applied in mathematics education to develop contextual and theory-based instructional designs. Fowler et al., (2023) emphasized that while DBR holds great potential for advancing educational theory and practice, many studies have yet to integrate this approach with both culturally and cognitively grounded instructional models. Therefore, this study aims to fill that gap by integrating the Van Hiele model and ethnomathematics within a DBR framework to trace and enhance students' cognitive progression in geometry learning. The novelty of this research lies in the combination of cognitive and cultural approaches within an iteratively developed instructional design (an Ethno–Van Hiele instructional scheme), with a specific focus on tracking students' geometric thinking development from visual to formal deductive levels.

This study expects students to show clear cognitive progression through the Van Hiele levels, particularly from visualization to analysis and informal deduction. By integrating ethnomathematics into the Van Hiele instructional model, the research aims to enhance students' geometric understanding and reasoning. The design-based approach is anticipated to produce an effective, culturally relevant instructional model that supports higher-level geometric thinking. Theoretically, this study contributes to expanding the Van Hiele's paradigm from a purely cognitive framework to a context based instructional framework. Practically, it provides a pedagogical model that can be adopted by secondary school teachers to foster students' geometric thinking progression through contextual, visual, and reflective learning activities.

METHOD

Research Type and Design

This study employed an Design Based-Research (DBR) approach grounded in the framework proposed by McKenney & Reeves (2012). DBR is a systematic inquiry aimed at designing, developing, and evaluating educational interventions to address complex problems in educational practice and theory (Cela-Ranilla et al., 2025; S. McKenney & Reeves, 2021). According to Akker et al. (2006), DBR can be categorized into three distinct types: (1) formative research, which focuses on the iterative development and refinement of educational interventions through continuous cycles of design and evaluation; (2) developmental research, which aims to generate new theoretical insights and deepen the understanding of design principles within educational contexts; and (3) engineering research, which emphasizes systematic design processes inspired by engineering principles to produce effective educational products or tools.The present study falls within the formative research type, as it seeks to develop and refine a learning design that integrates the Van Hiele Model of Instruction with ethnomathematics through two iterative learning cycles namely, the pilot class (cycle 1) and the actual class (cycle 2).

As articulated by McKenney & Reeves (2021), DBR follows an iterative and reflective process encompassing four key phases: analysis and exploration, design and construction, evaluation, and reflection. These phases operate cyclically, allowing for progressive refinement of both the theoretical constructs and the instructional design. The complete structure of these stages is illustrated in the figure 1.

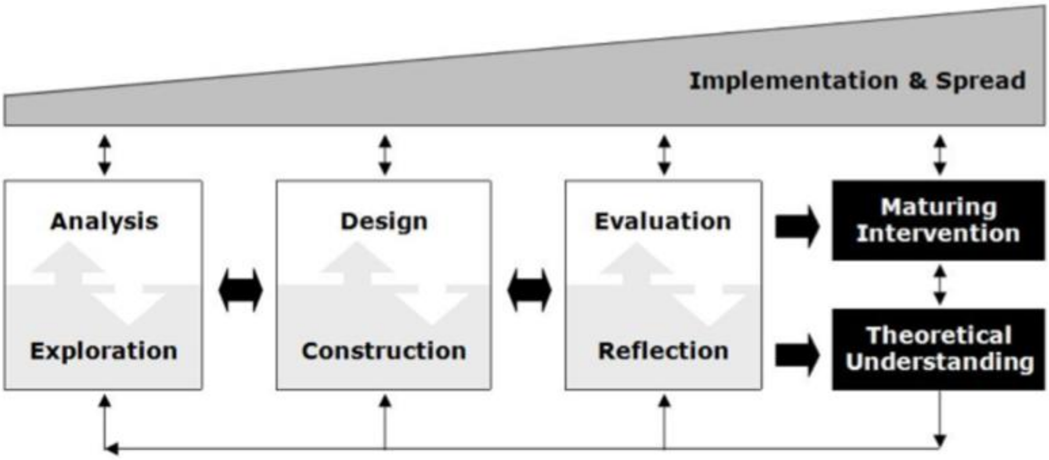

Figure 1. The McKenney & Reeves (2021) DBR Model

Participant

The selection of participants in this study employed a purposive sampling technique, which is a sampling method based on specific criteria determined by the researcher (Cohen et al., 2017 p218). The criteria used for participant selection included mathematics teachers who held a teaching certification, had at least ten years of teaching experience, and were willing to participate in the study. This selection aimed to obtain in-depth and relevant information concerning the focus of the research on teachers' professionalism practices in mathematics instruction at the junior high school level. Details of the participants are presented in Table 1.

Table 1. Participants of Research

Cycles	Location	Grade	n	The Teacher's Bacground	The Teacher Certificate	The Teacher's Experience	Agreement
1	SMPN 6 Tasikmalaya	8	34	Master of Mathematics Education	√	20 years	√
2	SMPN 13 Tasikmalaya	8	33	Bachelor of Mathematics Education	√	31 years	√

The participants in this study were two mathematics teachers who taught eighth-grade students at two public junior high schools in Tasikmalaya City, namely SMP Negeri 6 Tasikmalaya and SMP Negeri 13 Tasikmalaya. The teacher from SMP Negeri 6 Tasikmalaya taught eighth-grade students with a total of 34 learners, held a Master's degree in Mathematics Education, possessed a teaching certification, and had 20 years of teaching experience. Meanwhile, the teacher from SMP Negeri 13 Tasikmalaya also taught at the same grade level with 33 students, held a Bachelor's degree in Mathematics Education, possessed a teaching certification, and had 31 years of teaching experience.

Procedure and Instrument Development

Chen & Reeves (2020) conceptualize *DBR* as an iterative process that generates two principal outcomes: *a maturing intervention* and *theoretical understanding*. The maturation of an intervention emphasizes the systematic improvement of instructional practices through the sequential phases of analysis, design, and evaluation. Within this framework, the more iterative cycles are conducted, the more refined and contextually responsive the intervention becomes in addressing learning challenges. Conversely, theoretical understanding focuses on the development of educational theory through the stages of exploration, construction, and reflection**,** leading to a deeper scientific comprehension of how and why an intervention effectively operates. Hence, theoretical analysis and reflective inquiry are pivotal in generating novel conceptual insights that enrich the body of designed-based research (Dunn et al., 2019; Zhu & Zhang, 2023).

In practice, the McKenney & Reeves (2021) model is characterized by its dynamic and context-sensitive nature. Although the process appears sequential, the implementation of a design cannot be directly scaled to broader educational contexts. As illustrated in Figure 1, the model is applied progressively beginning with small-scale trials, followed by larger-scale implementations, and culminating in full-scale applications. Each phase necessitates iterative revisions informed by evaluative findings, including a reexamination of both the designed product and the contextual conditions initially analyzed. This simultaneous and recursive process ensures that the final intervention achieves a high degree of validity, usability, and readiness for classroom implementation (Fink et al., 2021; Tawfik et al., 2020).

This study adopts the DBR Model proposed by McKenney & Reeves (2021), which was subsequently adapted by the researcher into a customized research flow, as illustrated in Figure 2.

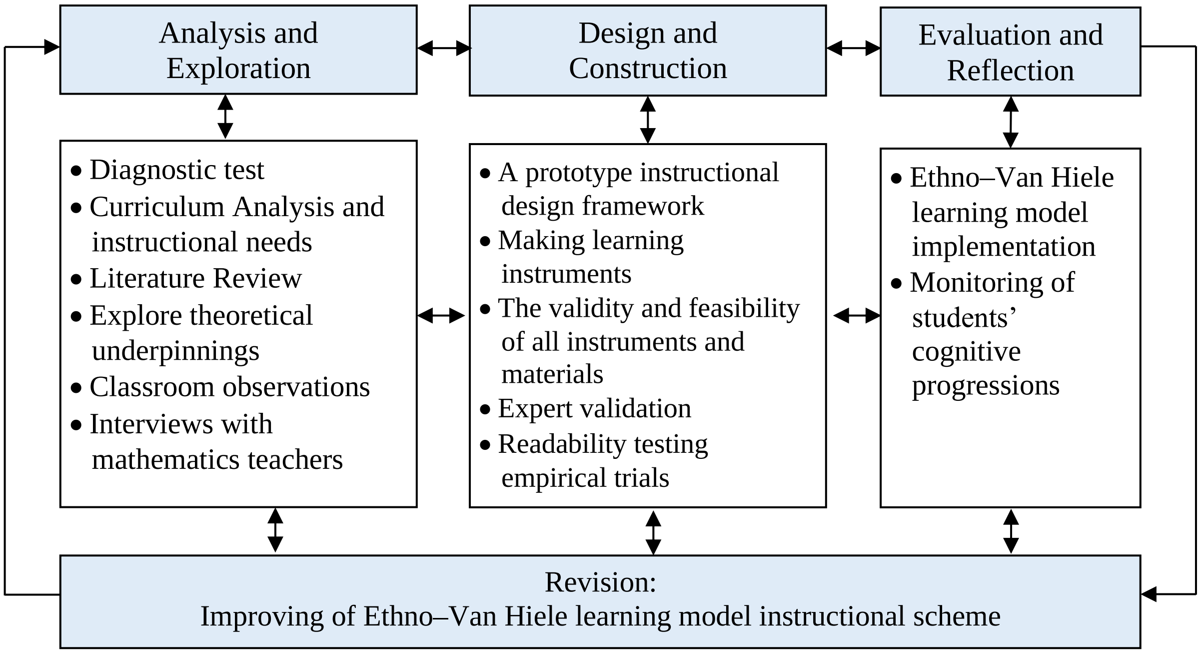

Figure 2. The Research Flow of Ethno-Van Hiele

Based on the research flow of ethno-van hiele (figure 2), the study commenced with the analysis and exploration phase, in which the researchers conducted a comprehensive preliminary study to identify students' difficulties in geometry, analyze curriculum and instructional needs, review relevant literature, explore theoretical underpinnings, and examine the school and learner contexts. The analysis of students' geometric difficulties was carried out through a diagnostic test on quadrilateral topics, developed according to Van Hiele's levels of geometric thinking. This assessment identified students' current levels of geometric understanding and their problem-solving challenges. The results served as the foundation for designing learning activities that optimized intervention at each level while appropriately integrating ethnomathematical contexts. The results of the diagnostic test are shown in Table 2.

Table 2. Result of Diagnostic Test in the Preliminary Trial Phase

Subtopic	Van Hiele Level	True answer	False Answer	Total	Achievement
Shapes and elements of quadrilaterals	Visualization	31	3	34	91%
Properties of quadrilaterals	Analysis	22	12		64%
Hierarchical relationships among quadrilaterals	Informal Deduction	5	19		15%
Axiomatic proof of quadrilaterals	Formal Deduction	3	31		8%

During the design and construction phase, the researchers developed a prototype instructional design framework that integrated the Van Hiele Model of Instruction with ethnomathematics, referred to as the *Ethno–Van Hiele model*. This instructional scheme guided the development of learning instruments, including lesson plans, student worksheets, and assessment tools**.** Additional instruments, such as observation sheets and interview protocols**,** were employed to ensure comprehensive data collection aligned with the study's objectives. The instruments in this study were integrated with Indonesian cultural artifacts. Examples of cultural artifacts used in this study are presented in Figure 3.

Figure 3. Cultural Artifacts Related to Quadrilaterals

Based on the selected cultural artifacts, the researchers made a table that maps the Van Hiele levels of geometric reasoning to relevant subtopics, learning objectives, task types, and corresponding worksheet or assessment items as in Table 3.

Table 3. Mapping Van Hiele Levels to Quadrilateral Learning Objectives with Cultural Artifacts

Van-Hiele Level	Subtopic	Learning Objective	Task Type	Assessment Item
Level 0	Identifying quadrilaterals in cultural patterns	Students can recognize and name geometric shapes embedded in cultural artifacts.	Visual identification	Identifying and name quadrilaterals in a woven motif from Tasikmalaya.
Level 1	Comparing properties of quadrilaterals	Students can analyze and compare properties of quadrilaterals.	Verbal explanation	Describing similarities and differences between rectangles and parallelograms in a rattan model.
Level 2	Calculating area using geometric formulas	Students can apply geometric formulas to calculate area of cultural artifacts.	Symbolic computation	Calculating the area of trapezoidal roof shapes from Kampung Naga and determine material needs.
Level 3	Proving geometric relationships	Students can prove geometric relationships using properties of quadrilaterals.	Verbal-symbolic reasoning	Prove that shafe of Kampung Naga Wall is a square or rhombus based on midpoint and perpendicularity conditions.
Level 4	Classifying quadrilaterals based on properties	Students can classify of quadrilaterals using formal geometric reasoning.	Deductive proof	Determine whether quadrilaterals is a rectangle, parallelogram, or rhombus based on given conditions.

To guarantee the validity and feasibility of all instruments and materials, expert validation, readability testing, and empirical trials were systematically conducted. Expert reviews involved content specialists and instructional media experts**,** while the readability test engaged five students to ensure that the materials were comprehensible and free from misconceptions. Instrument trials were conducted with students who had previously completed the geometry topic to establish empirical validity and reliability. The findings from these trials informed subsequent refinements of both instruments and teaching materials until they were deemed ready for classroom implementation.

The experts involved in this research included specialists in mathematics education, learning evaluation, and linguistics. The validation data for the research instruments are presented in the table 4.

Table 4. The Validation Result of Instrument

Validation Aspects	Instrumen			Validation Results
	Lesson plans	student worksheets	Assessment tools	
Content	0,91	0,90	0,88	High
Construct	0,88	0,89	0,92	High
Language	0,85	0,91	0,91	High

Based on Table 4, it can be seen that the validation results of the instruments have values above 0.8. This indicates that all instruments were categorized as highly valid.

In the evaluation and reflection phase, two cycles of field testing were conducted: (1) pilot class as cycle 1, and (2) actual class as cycle 2. In both cycle 1 and cycle 2, the learning activities were conducted twice per cycle. Prior to the implementation of the lessons, a briefing session was held with the teacher one day in advance to align perceptions between the teacher and the researcher. During the learning process, the teacher acted as a guide and facilitator, helping to direct the flow of activities and supporting students in achieving the learning objectives.

During the pilot class, the Ethno–Van Hiele learning model was implemented, and students' cognitive progression was closely monitored through classroom activities and artifacts such as worksheets and Van Hiele geometry tests. Post-session reflections identified the model's strengths and weaknesses, leading to iterative refinements aimed at enhancing instructional effectiveness before the main implementation.

In the main implementation phase, the improved *Ethno–Van Hiele* model was applied in actual classroom settings. Like the earlier stage, systematic evaluations were conducted on students' learning processes, observable behaviors, and tangible outputs, including worksheets and geometry test results. Following reflection and analysis, local theoretical constructions were formulated based on the refined instructional design and the emergent empirical findings.

Data Collection

Data collection techniques included tests, observations, interviews, and documentation. Tests comprising pre- and post-assessments were administered in each learning cycle and contained geometry problems aligned with the Van Hiele levels on the quadrilateral topic. Classroom observations were carried out using structured observation sheets, while semi-structured interviews were conducted outside instructional time to complement the observational data. Documentation served as a primary data source, encompassing students' learning artifacts and recorded classroom activities captured through video recordings.

Data Analysis

Data analysis followed the procedures of qualitative data analysis, encompassing data reduction, data display, and conclusion drawing/verification. During data reduction, collected data were coded and categorized to identify patterns of cognitive progression corresponding to Van Hiele's levels. Categorization facilitated the identification of thematic consistencies across stages, enabling the development of narrative summaries and meaningful excerpts for further interpretation. Data display involved organizing information systematically—through concept maps, network diagrams, and event timelines—to enhance clarity and interpretability. Finally, the process of conclusion drawing and verification entailed interpreting the meanings underlying the data, validating findings through triangulation and reflection, and deriving conclusive insights that directly addressed the research questions posed in this study.

RESULTS AND DISCUSSION

The study was conducted from August to September 2025 through three developmental phases: analysis and exploration, design and construction, and evaluation and reflection. During the analysis and exploration phase, a preliminary study was carried out to investigate students' difficulties in learning the topic of quadrilaterals. The preliminary study involved a diagnostic test administered to eighth-grade students at SMPN 6 Tasikmalaya (34 students). The detailed results of the diagnostic test are presented in Table 1. It summarizes students' understanding of quadrilateral subtopics based on Van Hiele levels. At Level 1 (Visualization), students achieved 91%, showing strong visual recognition of quadrilateral shapes, as reflected in questions involving cultural patterns like "bilik" and woven designs. At Level 2 (Analysis), achievement dropped to 64%, indicating moderate ability to analyze properties of quadrilaterals in batik motifs. Level 3 (Informal Deduction) and Level 4 (Formal Deduction) showed low achievement rates of 15% and 8%, respectively. These levels required logical reasoning and proof, which students struggled with, especially in tasks involving symbolic and verbal representations.

The next phase was design and construction. Based on the insights obtained from this diagnostic assessment, the researchers subsequently designed a geometry learning approach grounded in the Van Hiele theory, which was expected to facilitate students' cognitive progression in learning geometry. Through an in-depth review of relevant literature on the Van Hiele model and the significance of contextual grounding in students' initial understanding of geometry, a pedagogical solution emerged—namely, the integration of the Van Hiele Model of Instruction with ethnomathematical contexts, conceptualized within the Ethno–Van Hiele instructional framework, as illustrated in Table 5.

Table 5. The Instructional Scheme of the Ethno–Van Hiele Model

Content	Ethnomathematics	Van Hiele Levels of Geometric Thinking	Van Hiele's Model of Instruction and Ethnomathematics
- The types of quadrilaterals commonly introduced in secondary geometry include the square, rectangle, parallelogram, rhombus, kite, trapezium (or trapezoid), and the general quadrilateral - Properties of Quadrilaterals - Relationships and Hierarchical Positions Among Quadrilateral Elements	Forms of cultural ornaments and artifacts that encapsulate ethnomathematical elements encompass batik motifs, indigenous architectural structures, traditional games, and various other culturally rooted designs	Using visual perception and nonverbal reasoning to recognize and distinguish quadrilateral shapes (level 0) Analyzing the properties of quadrilaterals through measurement, folding, cutting, tracing, and the use of dynamic geometry software (level 1) Analyzing the relationships between properties and figures, formulating meaningful definitions, providing simple yet coherent arguments, and constructing logical	**Phase 1: Information** The teacher engages students in a guided discussion about geometric shapes and elements embedded in various cultural ornaments and artifacts. This dialogic activity aims to activate students' prior visual experiences and connect them with geometrical forms present in everyday cultural objects. **Phase 2: Guided Orientation** Students construct geometric figures based on their visualization of shapes derived from the selected cultural ornaments and artifacts. Working collaboratively, students are facilitated to explore and identify the distinctive properties of different types of quadrilaterals through hands-on and comparative analysis. **Phase 3: Explicitation** Students articulate the properties of quadrilaterals and formulate interrelationships among them based on the characteristics they have discovered. This phase emphasizes conceptual clarification and

Content	Ethnomathematics	Van Hiele Levels of Geometric Thinking	Van Hiele's Model of Instruction and Ethnomathematics
		maps and diagrams (level 2) Demonstrating deductive geometric proof, necessary and sufficient conditions, and the importance of definitions, postulates, and proof theorems (level 3). Understanding how mathematical systems are constructed involves the ability to employ diverse forms of reasoning and evidence, to describe the consequences of adding or removing axioms within a geometric system, and to comprehend both Euclidean and non-Euclidean geometries (level 4)	encourages students to verbalize their reasoning as a foundation for deeper geometric abstraction. **Phase 4: Free Orientation** Students are provided with practice problems that reflect the cognitive levels of geometric thinking as described in the Van Hiele model. These tasks allow learners to apply their understanding independently and demonstrate progression across geometric reasoning levels. **Phase 5: Integration** Students synthesize and summarize the properties of quadrilaterals, consolidating their understanding by drawing comprehensive conclusions from the learning activities. This phase aims to strengthen conceptual coherence and foster reflective awareness of geometric relationships within cultural and mathematical contexts

The scheme presented in Table 5 illustrates the systematically structured instructional design of the Ethno–Van Hiele Model, which is explicitly oriented toward the hierarchical levels of Van Hiele's geometric thinking. This scheme subsequently served as the foundation for developing the instructional instruments. The development of the lesson plan, student worksheet, and assessment instruments constituted a core component of the design and construction phase. In this phase, the lesson plan was constructed in the form of a teaching module that encapsulates the syntactical structure of the Ethno–Van Hiele model, consisting of five phases: information, guided orientation, explicitation, free orientation, and integration. In practice, each phase within the Ethno–Van Hiele syntax does not correspond directly or linearly to the Van Hiele levels but rather operates dynamically to foster students' gradual progression through these levels.

The evaluation assessment developed to measure students' attainment across the Van Hiele levels of geometric thinking. The assessment encompasses problems addressing all levels visualization, analysis, informal deduction, formal deduction, and rigor. Similar to the diagnostic assessment, it includes five subtopics on quadrilaterals: (1) forms and components of quadrilaterals (visualization level), (2) properties of quadrilaterals (analysis level), (3) hierarchical relationships among quadrilaterals (informal deduction level), (4) axiomatic proofs of quadrilaterals (formal deduction level), and (5) application of necessary and sufficient conditions within axiomatic proofs (rigor level).

The evaluation and reflection phase serves to systematically assess the validity, practicality, and effectiveness of the instructional scheme and its supporting instruments through two cycles of implementation an trial class (cycle 1) and a actual class (cycle 2). The cycle 1 was conducted with eighth-grade students at SMPN 6 Tasikmalaya on August 2025, while cycle 2 was carried out with eighth-grade students at SMPN 13 Tasikmalaya on September 2025.

The first cycle began with contextual problems rooted in ethnomathematical elements such as batik, woven mats, and traditional kites. These familiar cultural artifacts helped students transition from intuitive visualization to formal geometric reasoning. Students successfully identified quadrilaterals (visualization level), as shown in Figure 4.

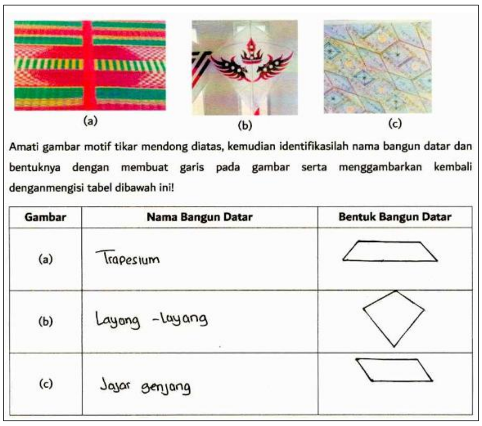

Figure 4. Students Identifying and Drawing Quadrilaterals

Students were able to identify and describe the properties of quadrilaterals (analysis level). However, their articulation of these properties appeared to be fragmented and largely derived from surface-level identification of visual representations. Consequently, students experienced difficulties in distinguishing between kites, trapezoids, and parallelograms based on their defining attributes. This limitation subsequently affected their understanding of geometric relationships in the following learning phases. A detailed example of students' responses is presented in Figure 5.

Figure 5. Example of Students' Responses

At the informal deduction phase, students experienced difficulties in understanding the hierarchical relationships among quadrilaterals, such as distinguishing whether a square is considered a rectangle. Although the teacher provided explanations, students' limited understanding of quadrilateral properties continued to hinder their progress. This affected their ability to reach the formal deduction and rigor levels, with only a few students achieving formal deduction and none reaching the rigor level. Therefore, the researchers revised the instructional instruments to be used in cycle 2. The revised components based on the results of cycle 1 included: (1) the instructional scheme, (2) cultural artifacts featuring a wider variety of quadrilateral shapes, and (3) the use of scaffolding questions.

The actual classroom implementation began with revsions to the instructional scheme and learning materials. These improvements were focused on addressing discrepancies between the pilot class and actual classroom activities. The teacher introduced the lesson by narrating the traditional weaving of tikar mendong from Tasikmalaya. Students were asked to identify quadrilaterals within the woven pattern image. Although they were able to name and draw several shapes, their recognition was limited to visually obvious forms, as shown in Figure 6.

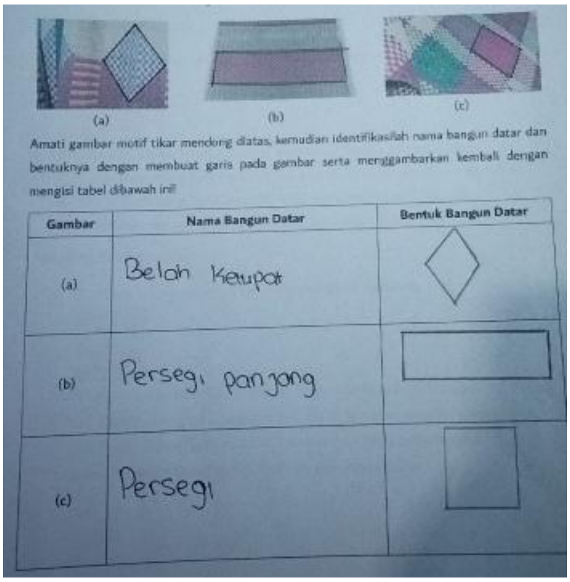

Figure 6. Students' Responses in Identifying Quadrilateral Shapes

To deepen understanding, the teacher applied a guided reinvention approach using scaffolding questions. This helped students discover less apparent quadrilaterals such as trapeziums, parallelograms, and kites through imaginative reasoning. The following excerpt illustrates the sequence of dialogic interaction during the guided reinvention process.

…………………………..

Teacher : "Are you certain that there are only three quadrilaterals in the figure?"

Student 1 : "Yes, Ma'am. I don't see any other quadrilateral."

Teacher : "Please take another careful look. Do you happen to find any trapezium?"

Student 2 : "Oh yes, Ma'am, I can see a trapezium in picture C. Is this the one you mean?" [pointing at the figure]

Teacher : "Yes, that's correct. Are there any other quadrilaterals you can find? I'll give a small reward if you can identify them."

Student 1 : "I found a parallelogram, Ma'am. And there's also a kite."

…………………………..

In the analysis phase, students explored quadrilateral properties through relational comparisons. Starting from the square as the most complex shape, they identified other quadrilaterals by recognizing which attributes were not satisfied. This approach helped students build hierarchical relationships among quadrilaterals, marking their transition into the informal deduction phase of the Van Hiele model. The following excerpt of the dialogue demonstrates how students' understanding of the properties of quadrilaterals evolved through reasoning grounded in the attributes of a square.

………………………..

Teacher : "Let us revisit the properties of a square that you have previously learned. If all sides are not equal in length, would the quadrilateral still be considered a square?"

Student 3 : "No, Ma'am. It would become a rectangle."

Teacher : "If the angles are no longer perpendicular to each other, would the quadrilateral still remain a square?"

Student 4 : "No, Ma'am. That would be a parallelogram"

………………………..

Students began to understand quadrilateral properties through relational comparisons, recognizing hierarchical connections among shapes like squares, rectangles, rhombuses, and trapeziums. This marks their entry into the informal deduction level of the Van Hiele model. Although not all students reached higher levels, some showed awareness of the need for logical reasoning to justify geometric truths.

At the formal deduction level, students started organizing knowledge using definitions, axioms, and theorems, shifting from visual to logical reasoning. In an ethnomathematical context, this transition reflects how students abstract geometric concepts from cultural experiences.

No students reached the rigor level because they had not yet fully mastered the logical structure and deductive system of geometry. Although some students began to show reflective awareness of the importance of logical reasoning, most remained at the informal or early formal deduction stages. They were not yet able to understand the internal consistency of axiomatic systems or question the necessary and sufficient conditions of geometric theorems, which are key characteristics of the rigor level in the Van Hiele model.

To trace the dynamics of students' cognitive development in understanding geometric concepts based on Van Hiele's theory, the researchers administered a Van Hiele geometry test after the completion of the entire instructional sequence. The result was illustrated in Table 6.

Table 6. Students' Cognitive Progression Across the Van Hiele Levels of Geometric Thinking

Subtopic	**Van Hiele Level**	**Achievement**	
		Cycle 1 (n = 34)	**Cycle 2 (n = 33)**
Shapes and elements of quadrilaterals	Visualization	85%	94%
Properties of quadrilaterals	Analysis	68%	82%
Hierarchical relationships among quadrilaterals	Informal Deduction	59%	70%
Axiomatic proof of quadrilaterals	Formal Deduction	41%	61%
The Use of Necessary and Sufficient Conditions in Axiomatic Proof	Rigor	0%	0%

In pilot class, students' cognitive progression reached its highest point at the visualization phase. However, the progression gradually declined as the Van Hiele level increased. A similar pattern emerged in actual class, where students again achieved their highest progression at the visualization phase, followed by a decrease in higher levels. Nevertheless, students' cognitive progression in actual class showed notable improvement compared to pilot class, largely due to the refinements made to the Ethno–Van Hiele model through the reconstruction of the learning scheme and instructional materials.

At the visualization phase, students demonstrated an understanding of quadrilateral shapes by visualizing them through batik and *tikar mendong* (woven mat) patterns. At this stage, they not only observed geometric models but also identified and marked various quadrilateral forms that aligned with their mental imagery. Some students were even able to adapt their mental representations to visualize quadrilateral forms embedded in batik and *mendong* motifs, even when these were not explicitly visible. Through this process, students successfully identified and represented nearly all quadrilateral models they had previously encountered. Ethnomathematical approaches in geometry can improve understanding of concepts through cultural context (Kyeremeh et al., 2023). This aligns with Fouze & Amit (2021), who, in their article *Teaching Geometry by Integrating Ethnomathematics of Bedouin Values*, demonstrated that integrating local cultural values into geometry instruction—when combined with the Van Hiele theory—can enhance students' understanding of geometric concepts. They emphasized that this approach makes learning more contextual and meaningful, particularly at the visualization and analysis levels.

At the analysis level, students began identifying the properties of quadrilaterals based on drawings derived from ethnomathematical models. In this stage, they were guided to explore geometric properties comprehensively in order to develop a clear understanding of the hierarchical relationships among quadrilaterals. The learning process deliberately began with the square, as it represents the most specific and complex form of quadrilateral. This approach aligns with the findings of Cybulski et al., (2024), who emphasized the importance of understanding hierarchical classification starting from the square. Similarly, Zeybek (2018) demonstrated that students' understanding of inclusion relations between quadrilaterals improved after instruction that emphasized hierarchical classification, such as beginning with squares. By understanding the square and its properties—many of which are not shared by other quadrilaterals—students were naturally led to recognize how the absence or alteration of certain properties generates other types of quadrilaterals. For example, when the sides are no longer equal, the figure can no longer be classified as a square but rather as a rectangle

During the informal deduction level, students began to establish relational connections among quadrilaterals. Initially, many struggled to accept that a square is a rectangle, reflecting a common misconception rooted in prototypical thinking. However, through sustained engagement with tasks emphasizing comprehensive property analysis and hierarchical classification, students gradually overcame these misconceptions. For example, when a student claimed that "a rectangle is a square," they recognized the logical inconsistency since a rectangle may have unequal sides, violating the defining property of a square. Conversely, when reasoning that "a square is a rectangle," students justified their claim by noting that a square meets all the criteria of a rectangle, with the added constraint of equal sides. This shift in reasoning illustrates a cognitive transition from descriptive (analytic) thinking to relational (informal deductive) thinking, as described in the Van Hiele model.

At this level, students began to connect semantic meaning (conceptual understanding) with syntactic structure (logical form), laying the foundation for formal deductive reasoning. This progression is supported by Arnal-Bailera & Manero (2024), who emphasized the importance of informal deduction in developing proof and classification skills within Van Hiele's geometric reasoning framework. Similarly, Zeybek (2018) found that students' understanding of inclusion relations among quadrilaterals improved significantly after instruction that emphasized hierarchical classification.

However, not all findings are unequivocally supportive. Sunardi et al., (2019) reported that students often experience anxiety during the informal deduction phase, particularly when required to justify geometric relationships without formal proof structures. This anxiety can hinder cognitive processing and reduce confidence in reasoning tasks. Moreover, Moru et al. (2020) observed that instructional gaps—such as skipping learning phases—can disrupt students' progression through Van Hiele levels, suggesting that informal deduction must be carefully scaffolded to be effective.

At the formal deduction level, students began organizing their geometric knowledge into logically structured systems, using axioms, definitions, and theorems as the foundation for constructing proofs. Unlike the informal deduction phase, which emphasized conceptual relationships and property implications, this phase revealed students' growing awareness that geometric truths could be demonstrated systematically and hierarchically through valid deductive arguments. A key cognitive shift observed at this level was the realization that conclusions no longer relied on visual observation or empirical reasoning, but instead on the internal logical coherence of the geometric system.

For instance, when proving that the diagonals of a rectangle bisect each other, students moved beyond visual cues and sequential reasoning. They applied congruence relationships between two pairs of triangles and logically deduced that the intersection of the diagonals divides them equally—thus proving the theorem rigorously. This transition aligns with Van Hiele's Level 4 (Deduction), where students begin to operate within a formal axiomatic system.

Arnal-Bailera & Manero (2024) emphasized that proof and definition are the most critical processes at this level, and that activities targeting formal deduction can reinforce earlier levels of geometric reasoning. Similarly, Maqoqa (2024) highlighted that Euclidean geometry, when taught with attention to logical structure and deductive reasoning, enhances students' critical thinking and problem-solving skills

The rigor level marks the highest level of geometric reasoning, where students understand and compare the logical structures of multiple geometric systems. However, this phase was not reached by the participants, as it requires abstract thinking typically developed in advanced education. Most junior high students remained in transition between informal and formal deduction, relying on visual and empirical reasoning. This aligns with Arnal-Bailera & Manero (2024), who noted that Van Hiele Level 5 is rarely observed at secondary level. Burger & Shaughnessy (1986) also found that formal deduction is uncommon among younger students, highlighting the difficulty of progressing to rigor. Conversely, Adeniji & Baker (2022) argued that Van Hiele strategies alone may not be sufficient to reach this phase without additional cognitive support.

CONCLUSION

This study demonstrates that students' cognitive progression in geometry significantly improved across all Van Hiele levels from cycle 1 to cycle 2, with a clear shift from visual and empirical reasoning toward relational and deductive thinking. Although the rigor phase was not yet attained, students exhibited progressive movement toward higher-order reasoning, supported by a refined instructional design. The Ethno–Van Hiele model played a central role in this development, guiding students to explore quadrilaterals through cultural artifacts, analyze their properties, understand hierarchical relationships, and construct formal proofs within an axiomatic framework. Three key design principles emerged: (1) relational analysis centered on the square, (2) a rich ethnomathematical context with varied representations, and (3) scaffolded discovery dialogue to support conceptual transitions. These findings affirm that geometric thinking is not a linear sequence of isolated skills but a dynamic cognitive system that can be expanded through culturally meaningful learning experiences. For educators, the study offers practical implications: (1) Select culturally relevant artifacts with geometric patterns. (2) Sequence tasks according to Van Hiele levels to support gradual cognitive development. (3) Use targeted prompts to elicit hierarchical reasoning, such as "What changes when a square becomes a rectangle?" or "Can all squares be considered rectangles?" This research contributes to a culture-based theory of geometry learning, integrating cognitive and contextual dimensions into a unified pedagogical framework that promotes deeper, humanistic, and contextually grounded mathematical understanding.

RECOMMENDATION

The findings of this study open up new avenues for the advancement of research and practice in geometry education. Future development can begin with extended studies that incorporate digital technologies, particularly to strengthen the analytical and informal deductive phases, enabling students to visualize the properties of quadrilaterals more concretely through applications such as GeoGebra, Geometry Sketchpad, and other dynamic geometry environments. Moreover, subsequent research should aim to explore broader mathematical competencies, including mathematical representation and spatial visualization skills, as integral components of cognitive progression. When the focus is confined solely to the levels of geometric thinking, learning achievements may become restricted to the formal deductive stage, whereas students' problem-solving models often exhibit richer and more diverse reasoning patterns that merit deeper investigation.

From the perspective of policy and practical implications, this study underscores the importance of ensuring teacher empowerment and sustainability through the establishment of collaborative spaces for reflection, replication, redesign, and dissemination of best teaching practices within schools and professional learning communities. Thus, the outcomes of this study are not merely conceptual innovations but serve as a catalyst for transformative praxis in future mathematics education.

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AUTHOR CONTRIBUTIONS STATEMENT

Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
Ipah Muzdalipah	✓	✓		✓	✓	✓	✓	✓	✓			✓	✓	✓
Sukirwan		✓	✓		✓	✓	✓	✓		✓	✓			✓
Sri Andayani		✓	✓		✓	✓	✓	✓		✓	✓			✓

CONFLICT OF INTEREST STATEMENT

The authors state that they have no conflict of interest related to the research, authorship, or publication of this article.

INFORMED CONSENT

We have obtained informed consent from all individuals included in this study.

ETHICAL APPROVAL

The research related to human use has been complied with all the relevant national regulations and institutional policies in accordance with the tenets of the Helsinki Declaration and has been approved by the authors' institutional review board or equivalent committee.

DATA AVAILABILITY

Data availability is not applicable to this paper as no new data were created or analyzed in this study.

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